Probing the Chiral Limit with Clover Fermions I: The Meson Sector

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– QCDSF Collaboration –

Outline

Lattice Simulations

Action

$$N_f = 2$$

$$S = S_G + S_F$$

$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re\,Tr}\, U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar\psi(x)\psi(x) - \kappa\,\bar\psi(x)U_\mu^\dagger(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) - \kappa\,\bar\psi(x)U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\, c_{SW}\, g\,\bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

$$\Updownarrow$$

$$\partial_\mu A_\mu^{\mathrm{imp}} = 2m_q P$$

Clover Fermions

Advantages

- Local
- Transfer matrix
- $O(a)$ improved
- Flavor symmetry

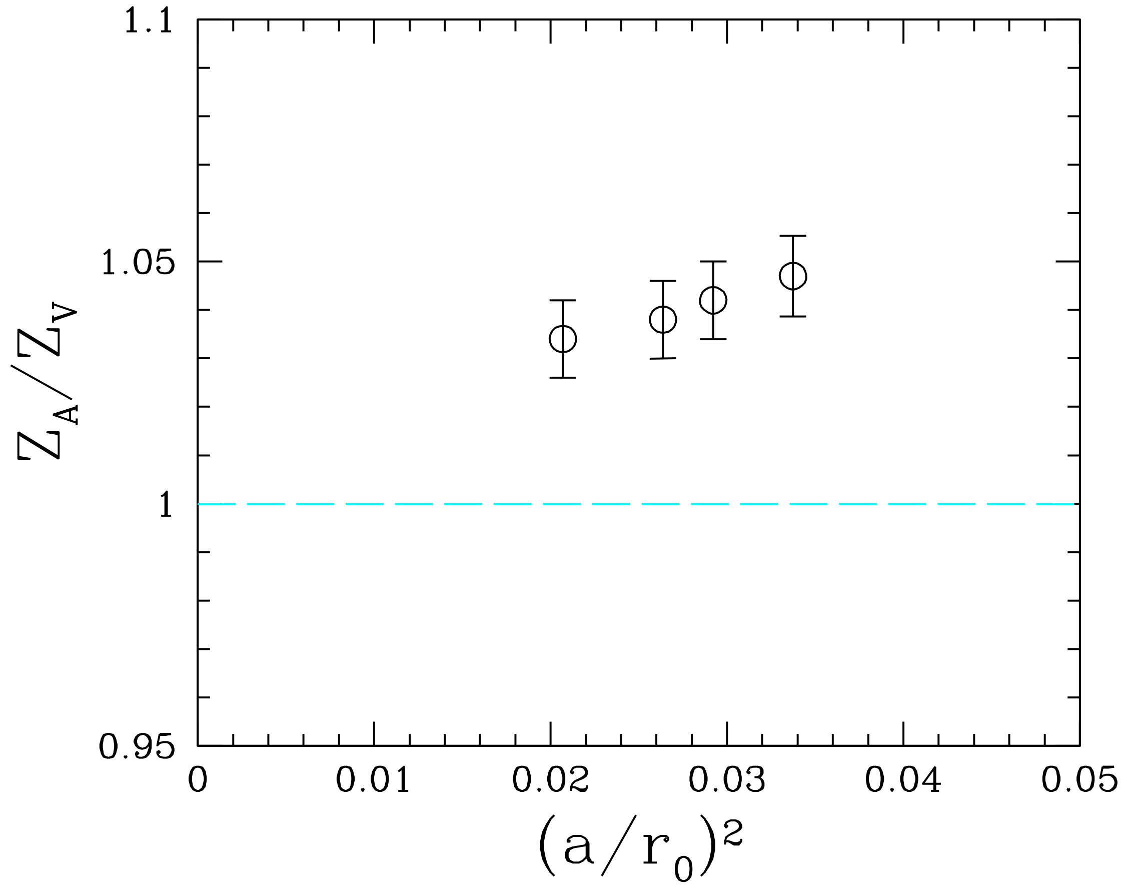
  Prerequisite to making contact with $SU(2)$ ChPT

  – Finite size corrections

  – Chiral extrapolation

  – Determination of low-energy constants
- Fast to simulate

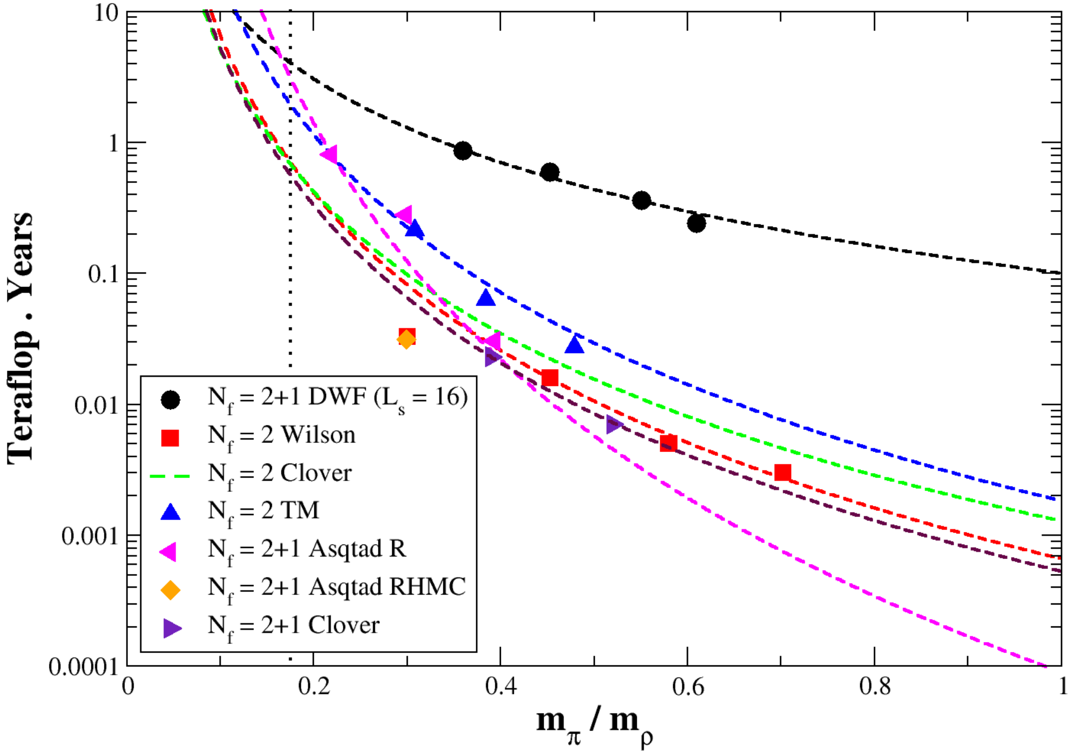
Chiral Symmetry ?



↑

NPRen

Cost of Simulation

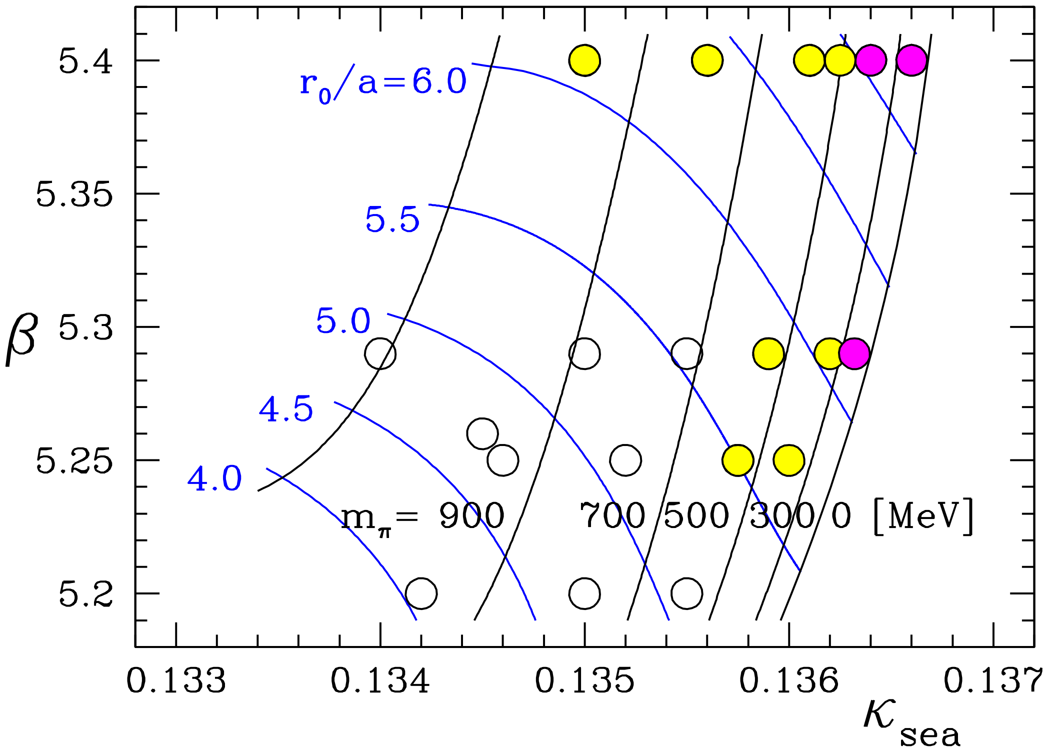


Clark

Lattices

β	κ_{sea}	Volume	a [fm]	m_{PS} [MeV]
5.20	0.13420	$16^3 \times 32$	0.11	1010
5.20	0.13500	$16^3 \times 32$	0.10	830
5.20	0.13550	$16^3 \times 32$	0.09	620
5.25	0.13460	$16^3 \times 32$	0.10	990
5.25	0.13520	$16^3 \times 32$	0.09	830
5.25	0.13575	$24^3 \times 48$	0.08	600
5.25	0.13600	$24^3 \times 48$	0.08	450
5.26	0.13450	$16^3 \times 32$	0.10	1010
5.29	0.13400	$16^3 \times 32$	0.10	1170
5.29	0.13500	$16^3 \times 32$	0.09	930
5.29	0.13550	$24^3 \times 48$	0.08	770
5.29	0.13550	$16^3 \times 32$		780
5.29	0.13550	$12^3 \times 32$		880
5.29	0.13590	$24^3 \times 48$	0.08	590
5.29	0.13590	$16^3 \times 32$		630
5.29	0.13590	$12^3 \times 32$		870
5.29	0.13620	$24^3 \times 48$	0.08	400
5.29	0.13632	$32^3 \times 64$	0.08	340
Not yet analyzed $\longrightarrow$ 5.29	0.13632	$40^3 \times 64$	0.08	290
5.40	0.13500	$24^3 \times 48$	0.08	1040
5.40	0.13560	$24^3 \times 48$	0.07	840
5.40	0.13610	$24^3 \times 48$	0.07	630
5.40	0.13625	$24^3 \times 48$	0.07	530
5.40	0.13640	$24^3 \times 48$	0.07	440
5.40	0.13640	$32^3 \times 64$	0.07	440
Not yet analyzed $\longrightarrow$ 5.40	0.13660	$32^3 \times 64$	0.07	280

Coverage



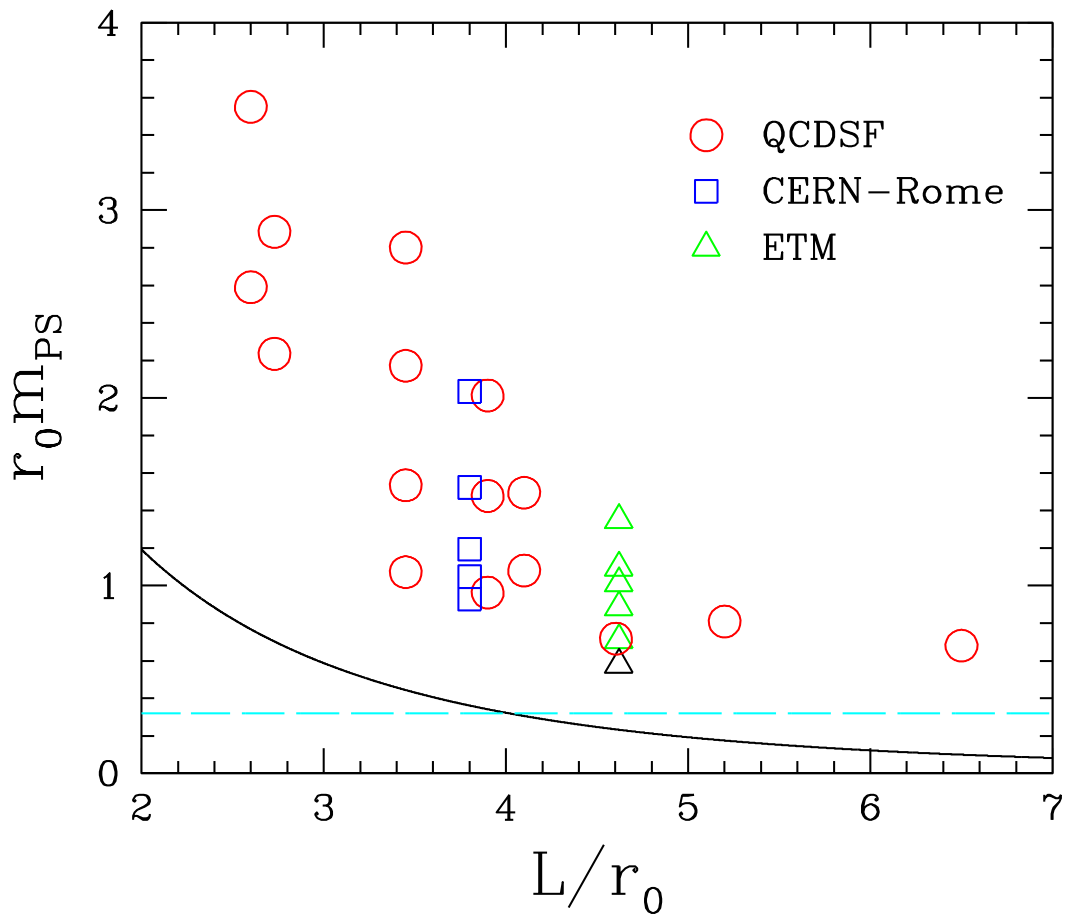
← $a = 0.07$ fm

← $a = 0.08$ fm

● $24^3\,48$ ● $32^3\,64 + 40^3\,64$

For gauge field sampling we use 'ordinary' HMC algorithm with Hasenbusch integration + 3 time scales

Landscape



Minimal pion mass : $m_{PS}(L) = \frac{3}{2 f_0^2 L^3} \left(1 + \frac{2}{4\pi f_0^2 L^2} 2.837 \right)^{-1}$

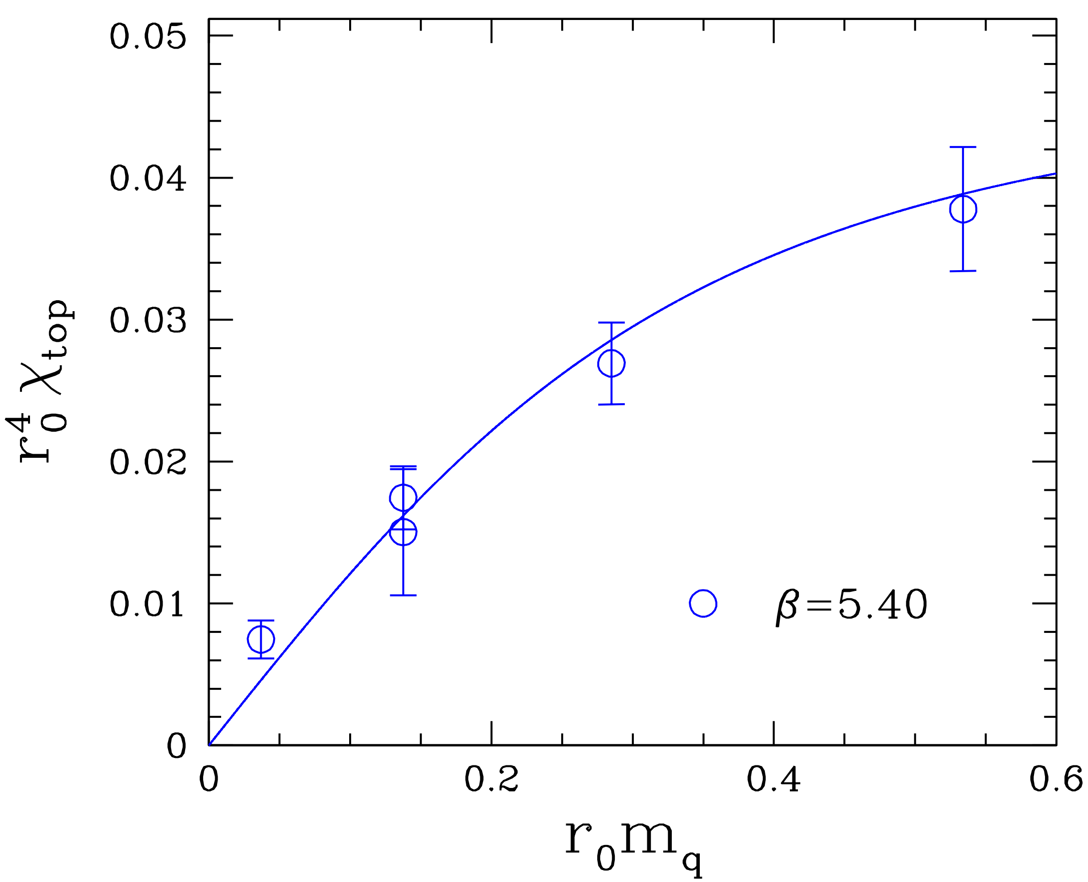
Leutwyler

Hasenfratz & Niedermayer

Effect of Unquenching ?

$$\chi_{\rm top} \equiv \frac{\langle Q^2 \rangle}{V} = \frac{\Sigma\, m_q}{2}$$

Vector Ward Identity √



$$\left(\frac{1}{\chi_{\rm top}}\right)^2 = \left(\frac{2}{\Sigma\, m_q}\right)^2 + \left(\frac{1}{\chi_{\rm top}^{\infty}}\right)^2$$

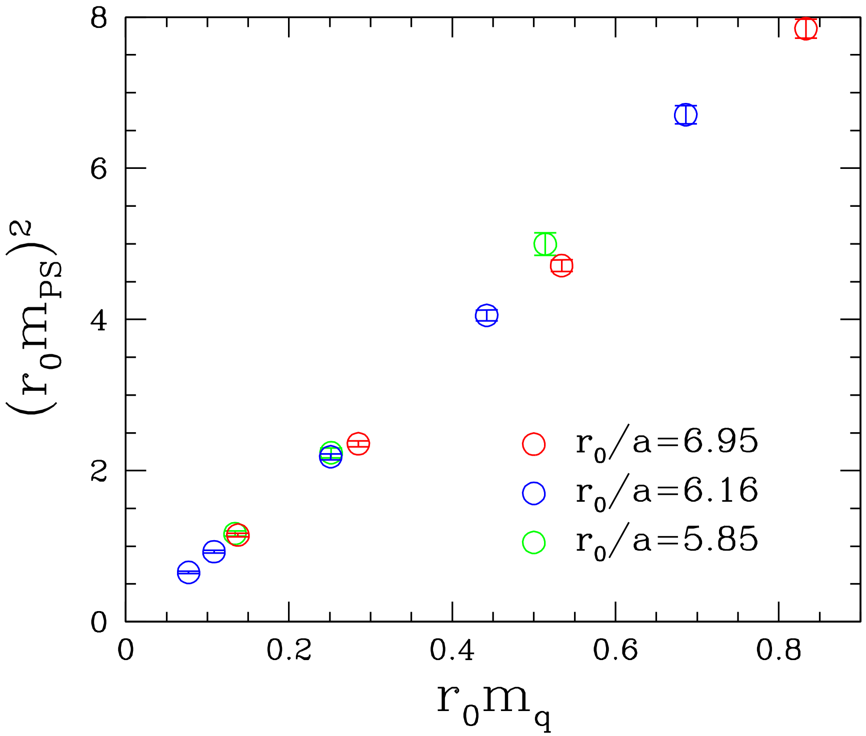
Dürr

$\Sigma^{\overline{MS}}(2\,\text{GeV}) = [276(12)\ \text{MeV}]^3$

Pion Sector

Pion Mass

Raw data



↑

NPRen

NLO

$$m_{PS}^2 = m_0^2 \left[1 - \frac{1}{2} x\, \hat{l}_3 + O(x^2) \right]$$

$$\frac{m_{PS} - m_{PS}(L)}{m_{PS}} = - \sum_{|\vec{n}| \neq 0} \frac{x}{2\lambda} \left[I_{m_{PS}}^{(2)}(\lambda) + x I_{m_{PS}}^{(4)}(\lambda) \right]$$

Colangelo, Dürr & Haefeli

$$m_0^2 = \frac{2\Sigma m_q}{f_0^2}\,, \quad x = \frac{m_0^2}{16\pi^2 f_0^2}\,, \quad \lambda = m_{PS} |\vec{n}| L$$

$$\hat{l}_i = \ln \frac{\Lambda_i^2}{m_0^2}$$

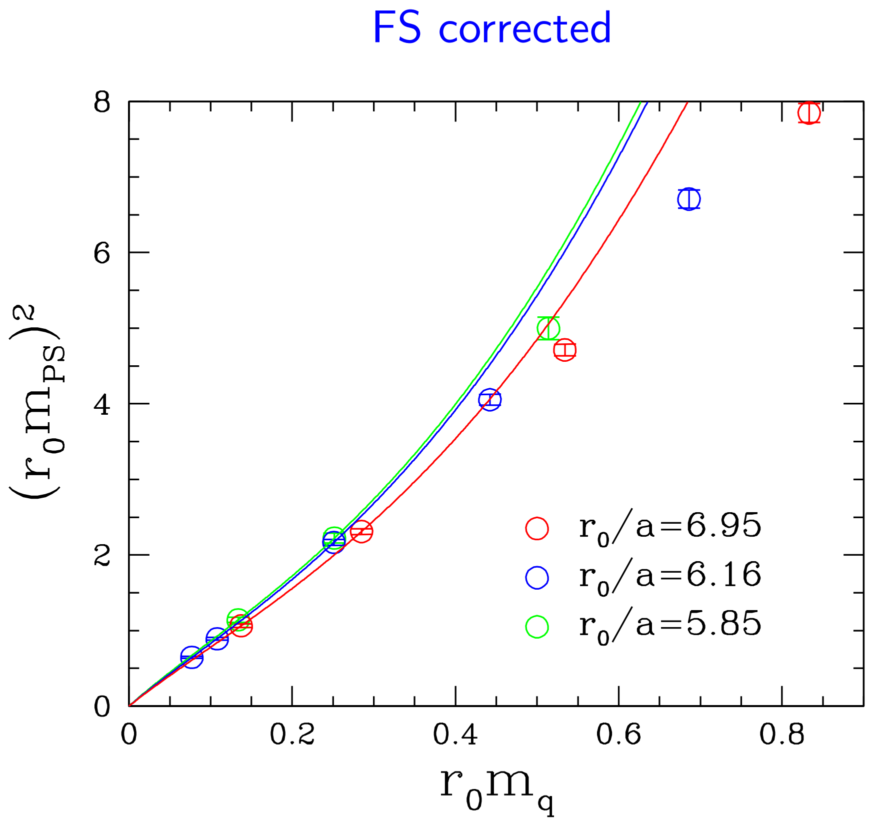
$$I^{(2)}_{m_{PS}}(x) = -B^0(x)$$

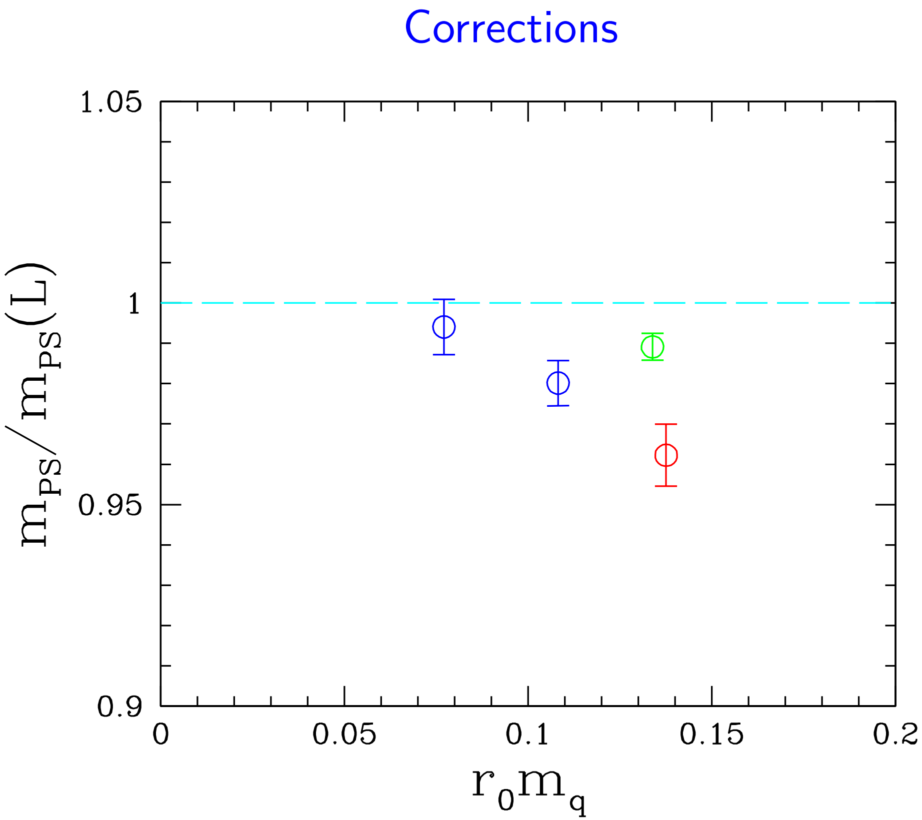
$$I^{(4)}_{m_{PS}}(x) = \left(-\frac{55}{18} + 4\bar{l}_1 + \frac{8}{3}\bar{l}_2 - \frac{5}{2}\bar{l}_3 - 2\bar{l}_4\right) B^0(x) + \left(\frac{112}{9} - \frac{8}{3}\bar{l}_1 - \frac{32}{3}\bar{l}_2\right) B^2(x) + S^{(4)}_{m_{PS}}(x)$$

$$S^{(4)}_{m_{PS}}(x) = \frac{13}{3} g_0 B^0(x) - \frac{1}{3}\left(40 g_0 + 32 g_1 + 26 g_2\right) B^2 + \cdots$$

$$B^0(x) = 2K_1(x)\,, \quad B^2(x) = 2K_2(x)/x\,, \quad \bar{l}_i = \ln\frac{\Lambda_i^2}{m_{PS}^2}$$

Λ_i , g_i from hep-lat/05030142

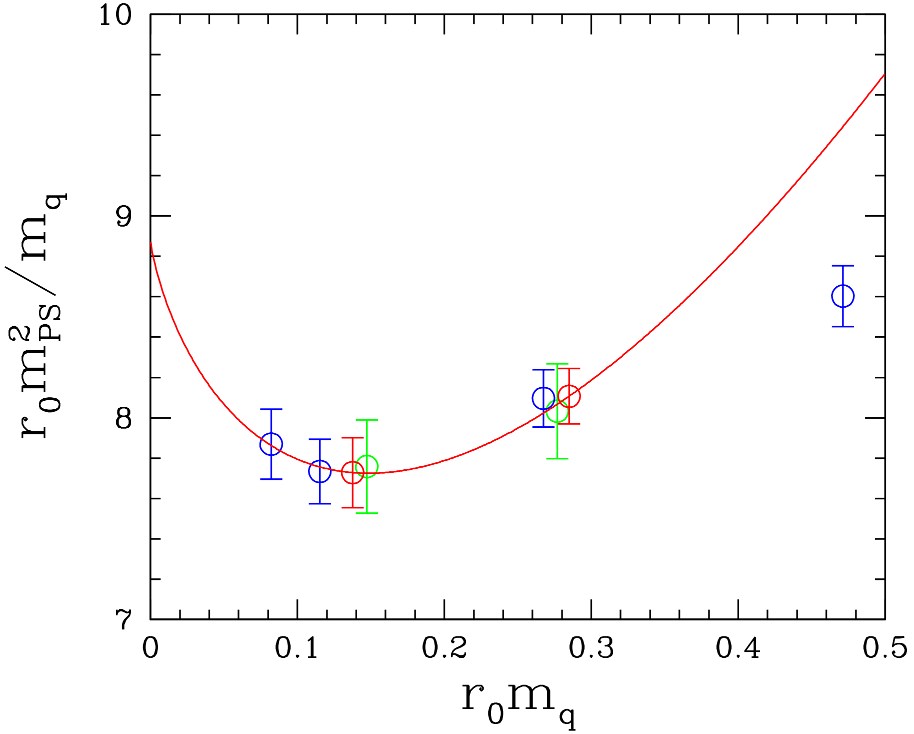


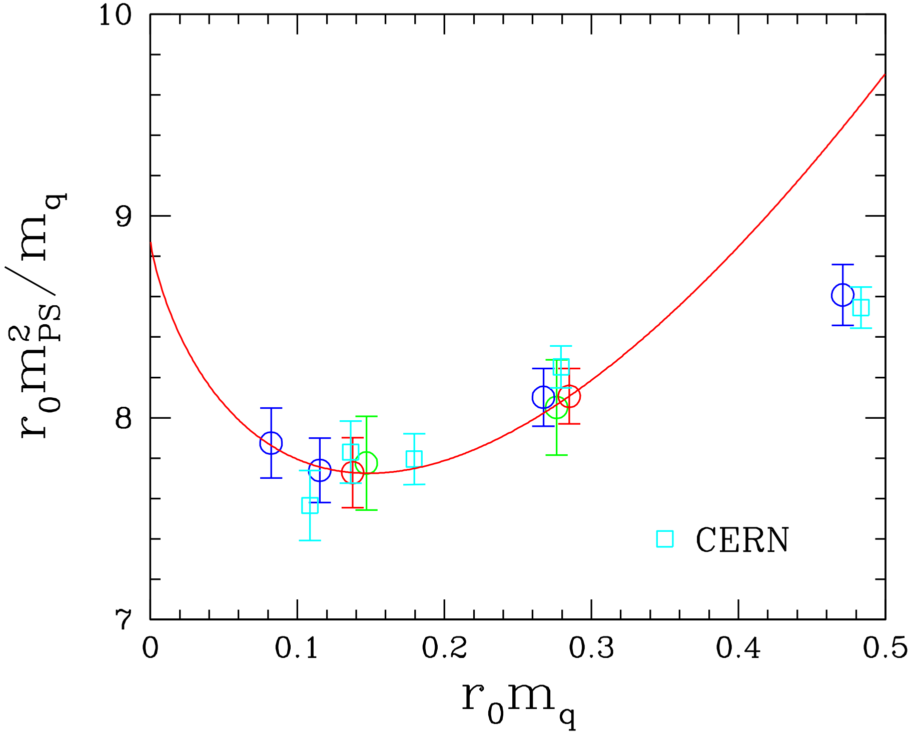


$r_0 f_0 = 0.179(2)\,, \quad r_0\Lambda_3 = 1.82(7)$

$\langle\bar{\psi}\psi\rangle^{\overline{MS}} = -[273(12)\ \text{MeV}]^3$

Chiral Logs ?

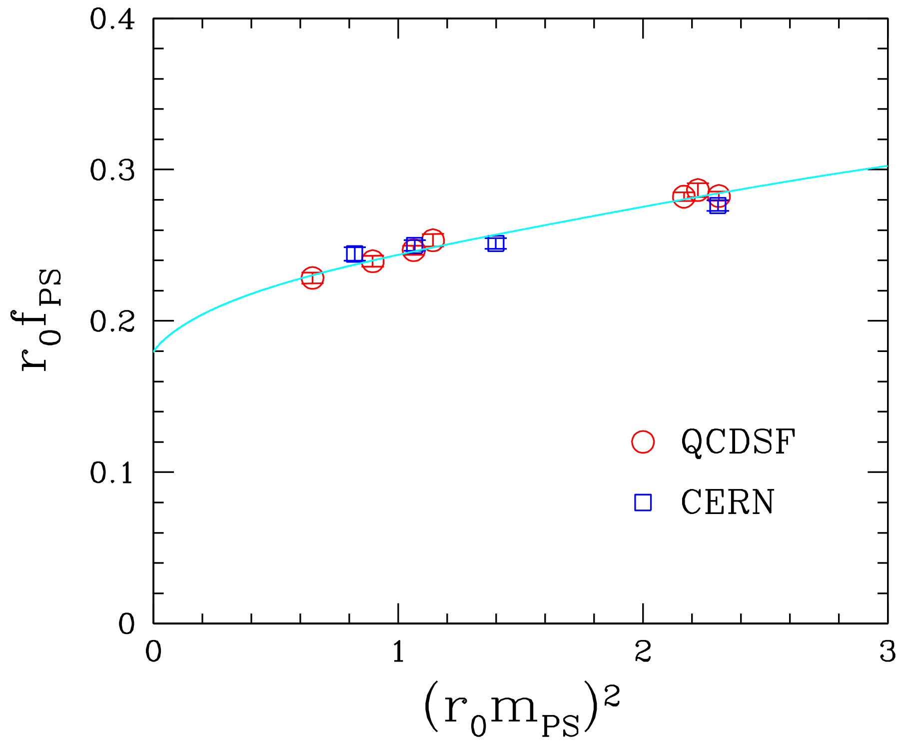




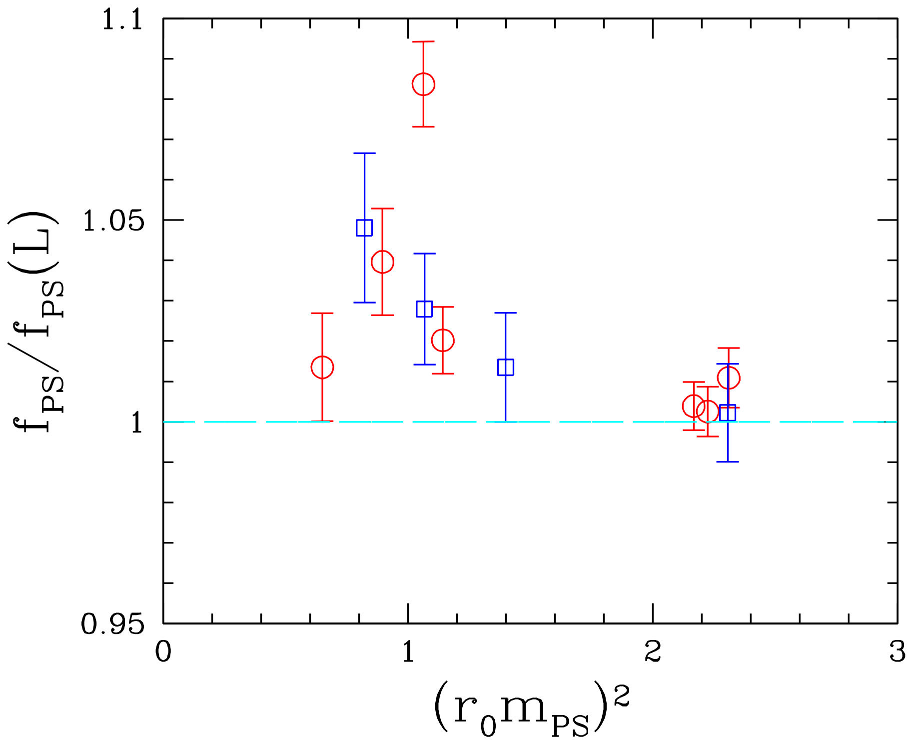
Corrected for $O(a^2)$ effects

Pion Decay Constant

FS corrected



Corrections



$$f_{PS} = f_0 \left[1 + x\, \hat{l}_4 + O(x^2) \right]$$

$$\frac{f_{PS} - f_{PS}(L)}{f_{PS}} = \sum_{|\vec{n}| \neq 0} \frac{x}{\lambda} \left[I^{(2)}_{f_{PS}}(\lambda) + x I^{(4)}_{f_{PS}}(\lambda) \right]$$

$r_0 f_0 = 0.179(2) \quad r_0 \Lambda_4 = 3.32(6)$

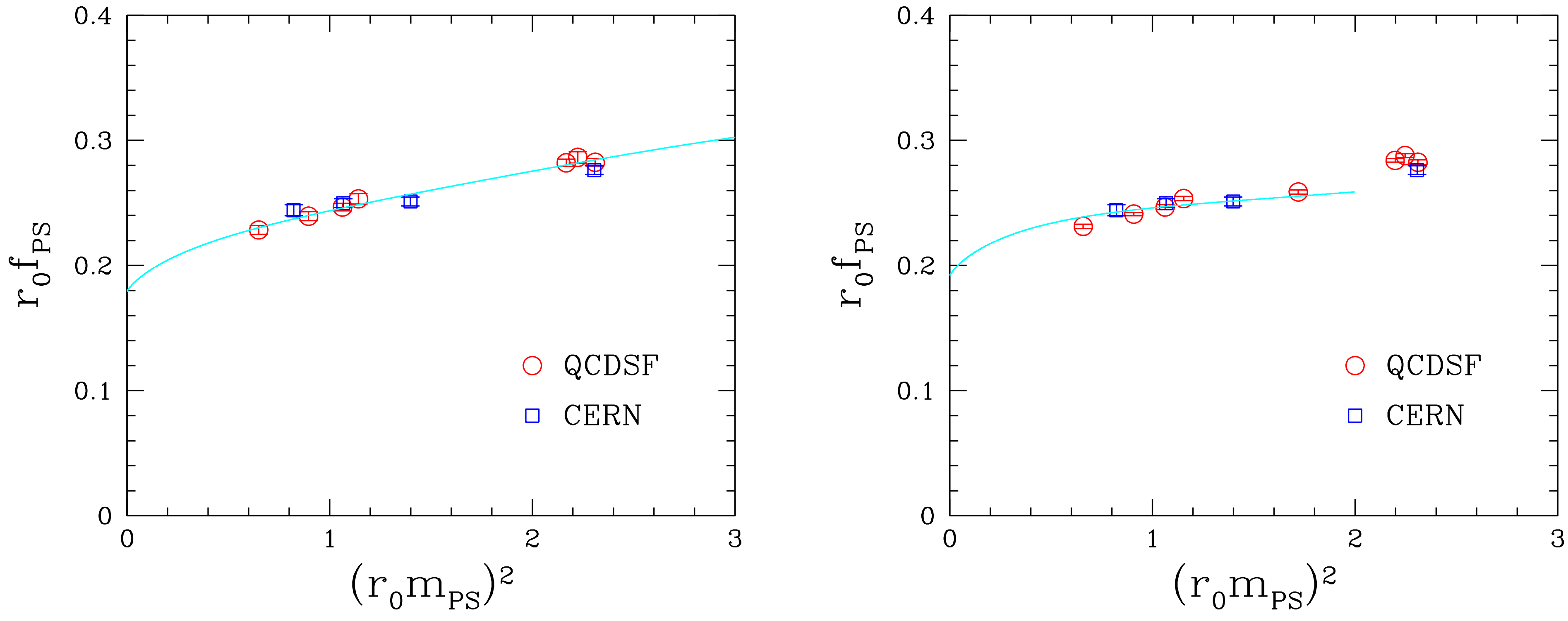
$f_{PS} \leftarrow$ NPRen

$$I^{(2)}_{f_{PS}}(x) = -2B^0(x)$$

$$I^{(4)}_{f_{PS}}(x) = \left(-\frac{7}{9} + 2\bar{l}_1 + \frac{4}{3}\bar{l}_2 - 3\bar{l}_4\right) B^0(x) + \left(\frac{112}{9} - \frac{8}{3}\bar{l}_1 - \frac{32}{3}\bar{l}_2\right) B^2(x) + S^{(4)}_{f_{PS}}(x)$$

$$S^{(4)}_{f_{PS}}(x) = \frac{1}{6}\,(8g_0 - 13g_1)\,B^0(x) - \frac{1}{3}\,(40g_0 - 12g_1 - 8g_2 - 13g_3)\,B^2 + \cdots$$

Colangelo, Dürr & Haefeli

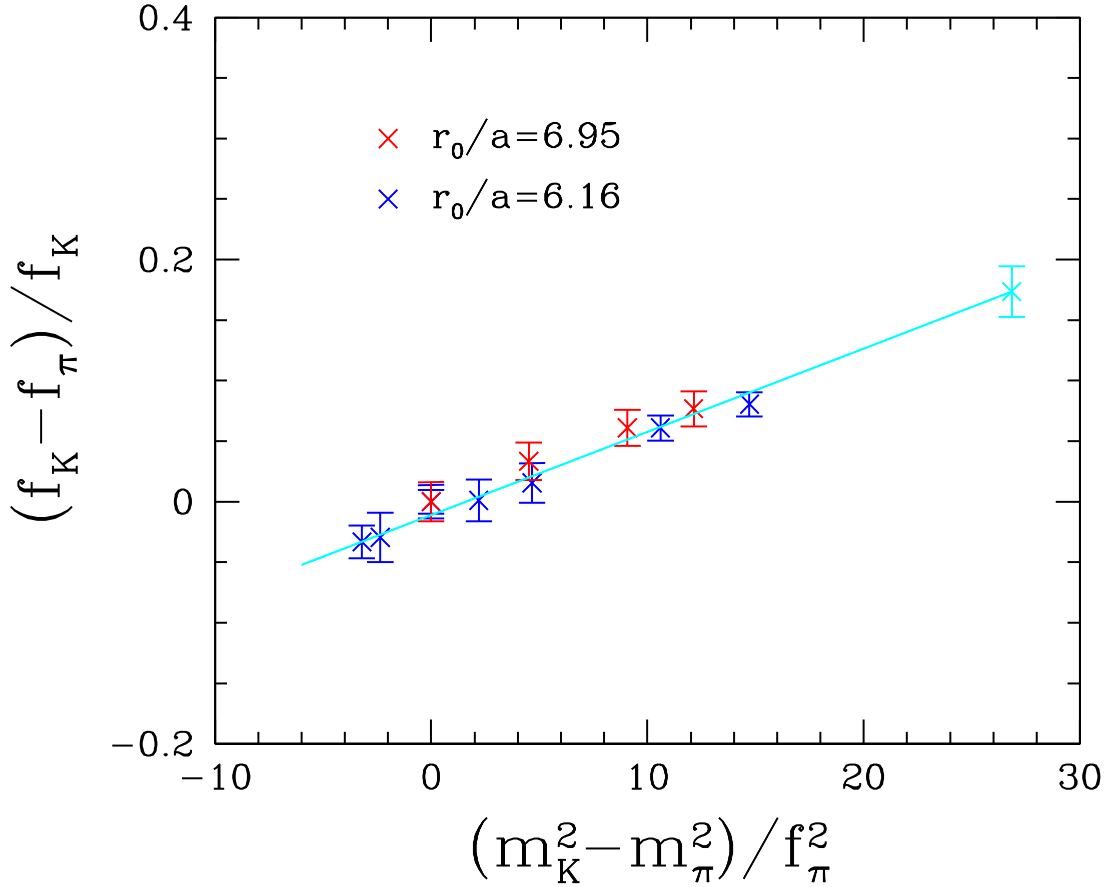


$r_0 f_0 = 0.179(2) \quad r_0\Lambda_4 = 3.32(6)$

$r_0 f_0 = 0.192(3) \quad r_0\Lambda_4 = 3.32(6)$

Kaon Decay Constant

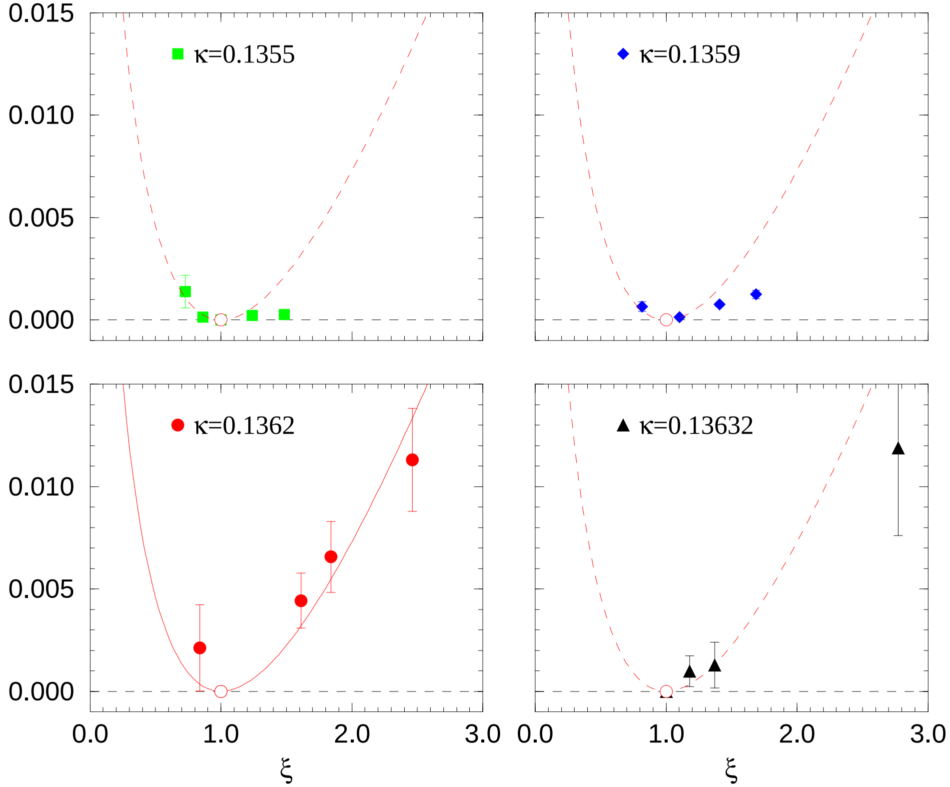
$$\frac{f_K - f_\pi}{f_K} = \frac{4L_5(m_K^2 - m_\pi^2)/f_\pi^2}{1 + 4L_5(m_K^2 - m_\pi^2)/f_\pi^2} + \text{logs}$$



$f_K/f_\pi = 1.21(3)$ $\quad L_5 = 0.0017$

Partially Quenched/Nondegenerate

$$m_{PS} \equiv m_{PS}^{SS} \rightarrow m_{PS}^{AB}\,, \quad f_{PS} \equiv f_{PS}^{SS} \rightarrow f_{PS}^{AB}\,, \qquad A, B \in \{V, S | V \neq S\}$$



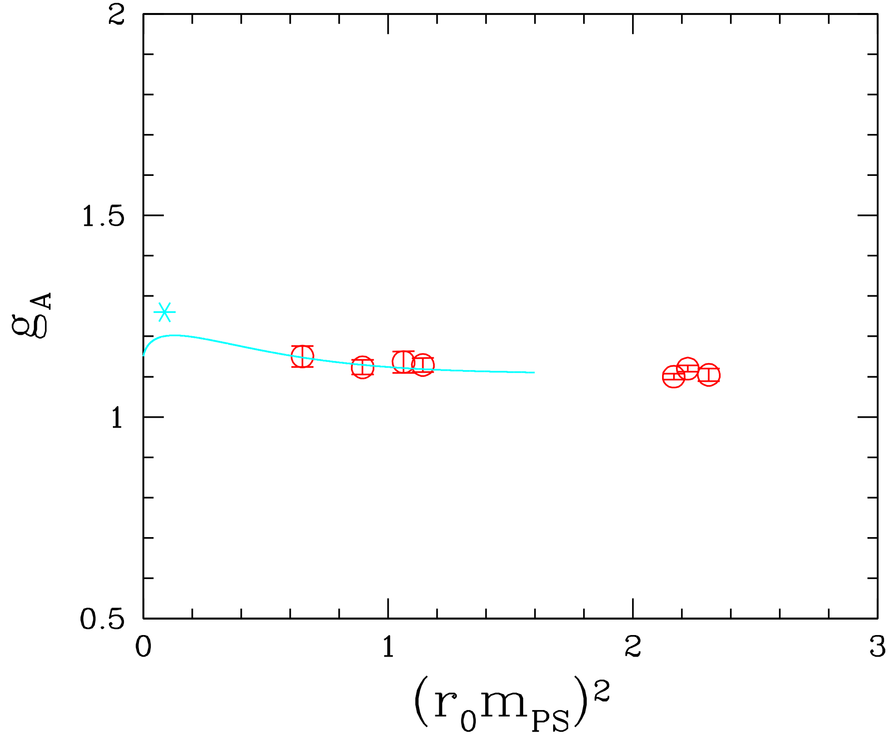
$$\hat{R} \equiv \frac{R}{m_{PS}^{SS\,2}} = \frac{f_{PS}^{VS}}{m_{PS}^{SS\,2}\sqrt{f_{PS}^{VV} f_{PS}^{SS}}} = -\frac{1}{8(4\pi r_0 f_0)^2}\left(\ln\frac{m_{PS}^{VV\,2}}{m_{PS}^{SS\,2}} - \frac{m_{PS}^{VV\,2}}{m_{PS}^{SS\,2}} + 1\right)\,, \quad \xi = \frac{m_{PS}^{VV\,2}}{m_{PS}^{SS\,2}}$$

Sharpe

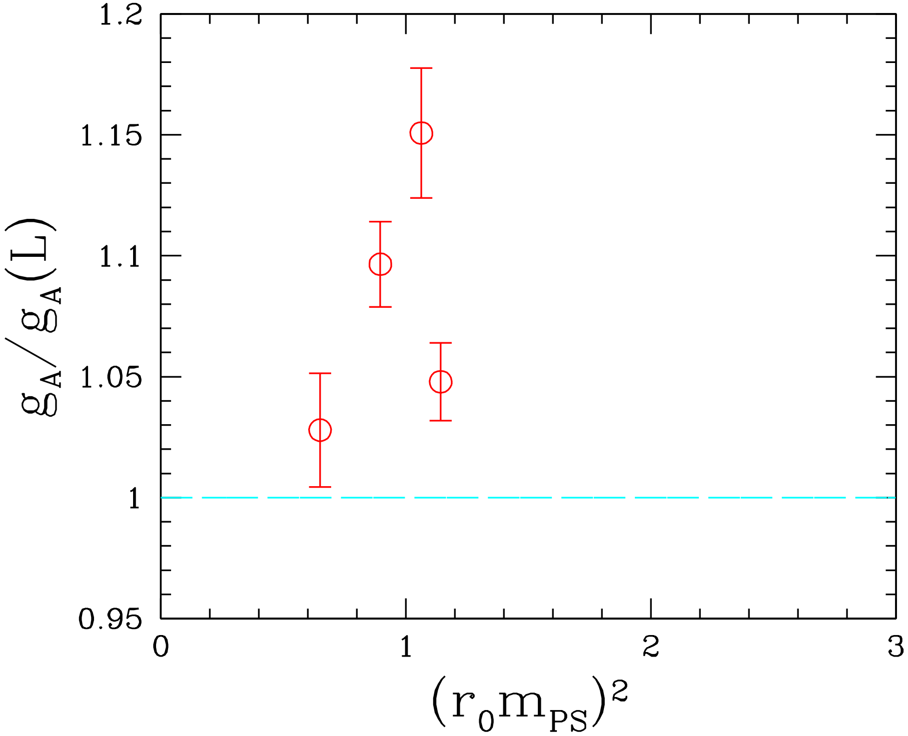
Setting the Scale

Axial Coupling

FS corrected



Corrections



$r_0 f_0 = 0.179(2)$ $g_A^0 = 1.15$

$g_1 = 4.0$

$c_A = 1.5$

$$
\begin{aligned}
g_A = g_A^0 &+ \left[4\, B_9^r(\lambda) - 8\, g_A^0 B_{20}^r(\lambda) - \frac{g_A^{0\,3}}{16\pi^2 f_0^2} - \frac{25 c_A^2 g_1}{324\pi^2 f_0^2} + \frac{19 c_A^2 g_A^0}{108\pi^2 f_0^2}\right] m_{PS}^2 \\
&- \frac{m_{PS}^2}{4\pi^2 f_0^2}\left[g_A^{0\,3} + \frac{1}{2}\, g_A^0\right] \ln\frac{m_{PS}}{\lambda} + \frac{4 c_A^2 g_A^0}{27\pi \Delta_0 f_0^2}\, m_{PS}^3 \\
&+ \left[25 c_A^2 g_1 \Delta_0^2 - 57 c_A^2 g_A^0 \Delta_0^2 - 24 c_A^2 g_A^0 m_{PS}^2\right] \frac{\sqrt{m_{PS}^2 - \Delta_0^2}}{81\pi^2 f_0^2 \Delta_0} \arccos\frac{\Delta_0}{m_{PS}} \\
&+ \frac{25 c_A^2 g_1 \left(2\Delta_0^2 - m_{PS}^2\right)}{162\pi^2 f_0^2} \ln\frac{2\Delta_0}{m_{PS}} + \frac{c_A^2 g_A^0 \left(3 m_{PS}^2 - 38\Delta_0^2\right)}{54\pi^2 f_0^2} \ln\frac{2\Delta_0}{m_{PS}} + \mathcal{O}(m_{PS}^4)
\end{aligned}
$$

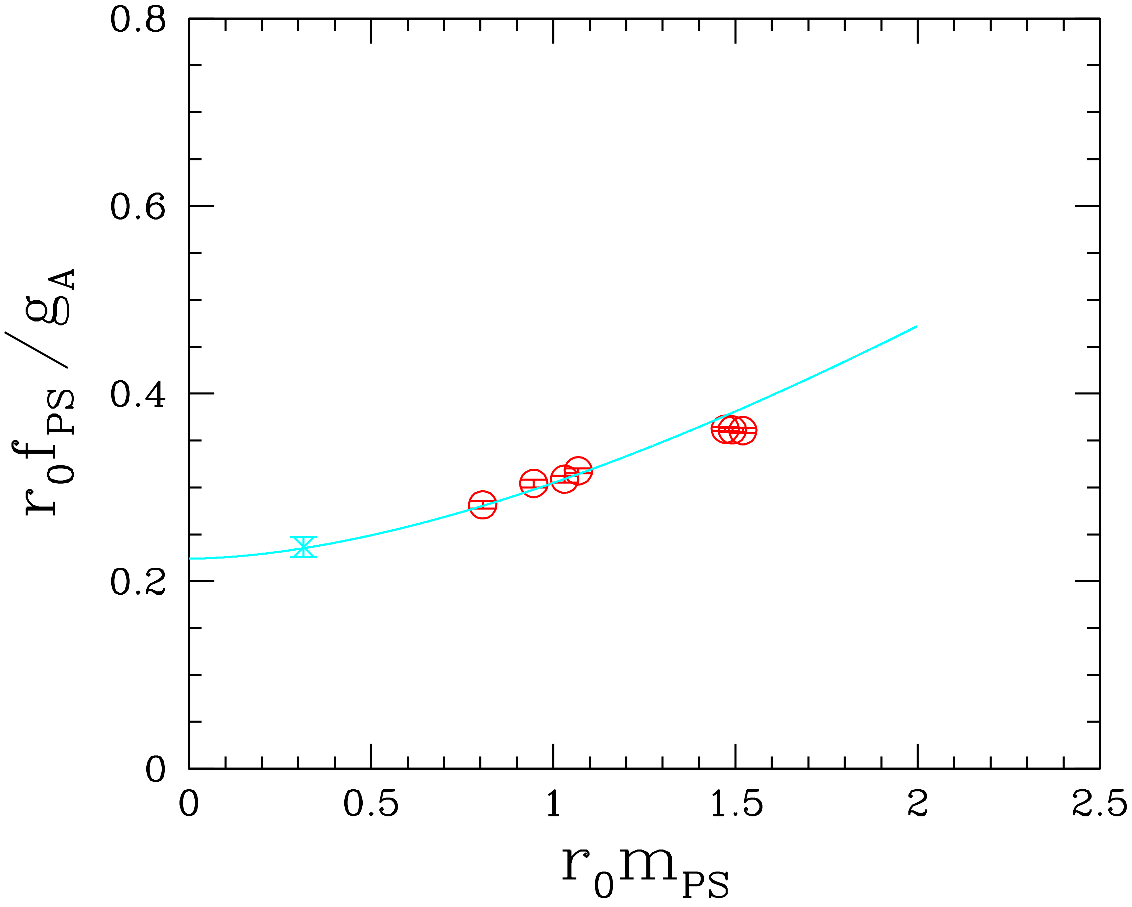
$$\Delta_0 = 0.271 \text{ GeV}$$

$$
\begin{aligned}
g_A - g_A(L) \; &= \frac{g_A^0 m_{PS}^2}{4\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \frac{K_1(\lambda)}{\lambda} - \frac{{g_A^0}^3 m_{PS}^2}{6\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \left[K_0(\lambda) - \frac{K_1(\lambda)}{\lambda} \right] \\
&- \left(\frac{25 c_A^2 g_1}{81\pi^2 f_0^2} - \frac{c_A^2 g_A^0}{\pi^2 f_0^2} \right) \int_0^\infty dy \, y \sum_{|\vec{n}|\neq 0} \left[K_0(\lambda_f) - \frac{\lambda_f}{3} K_1(\lambda_f) \right] \\
&- \frac{8 c_A^2 g_A^0}{27\pi^2 f_0^2} \int_0^\infty dy \sum_{|\vec{n}|\neq 0} \frac{f(m_{PS}, y)^2}{\Delta_0} \left[K_0(\lambda_f) - \frac{K_1(\lambda_f)}{\lambda_f} \right] \\
&+ \frac{4 c_A^2 g_A^0}{27\pi f_0^2} \frac{m_{PS}^3}{\Delta_0} \sum_{|\vec{n}|\neq 0} \frac{\mathrm{e}^{-\lambda}}{\lambda} + \mathcal{O}(m_{PS}^4)
\end{aligned}
$$

$$
f(m_{PS}, y) = \sqrt{m_{PS}^2 + y^2 + 2y\Delta_0}\,, \; \lambda_f = f(m_{PS}, y)|\vec{n}|L
$$

QCDSF

Determination of Scale

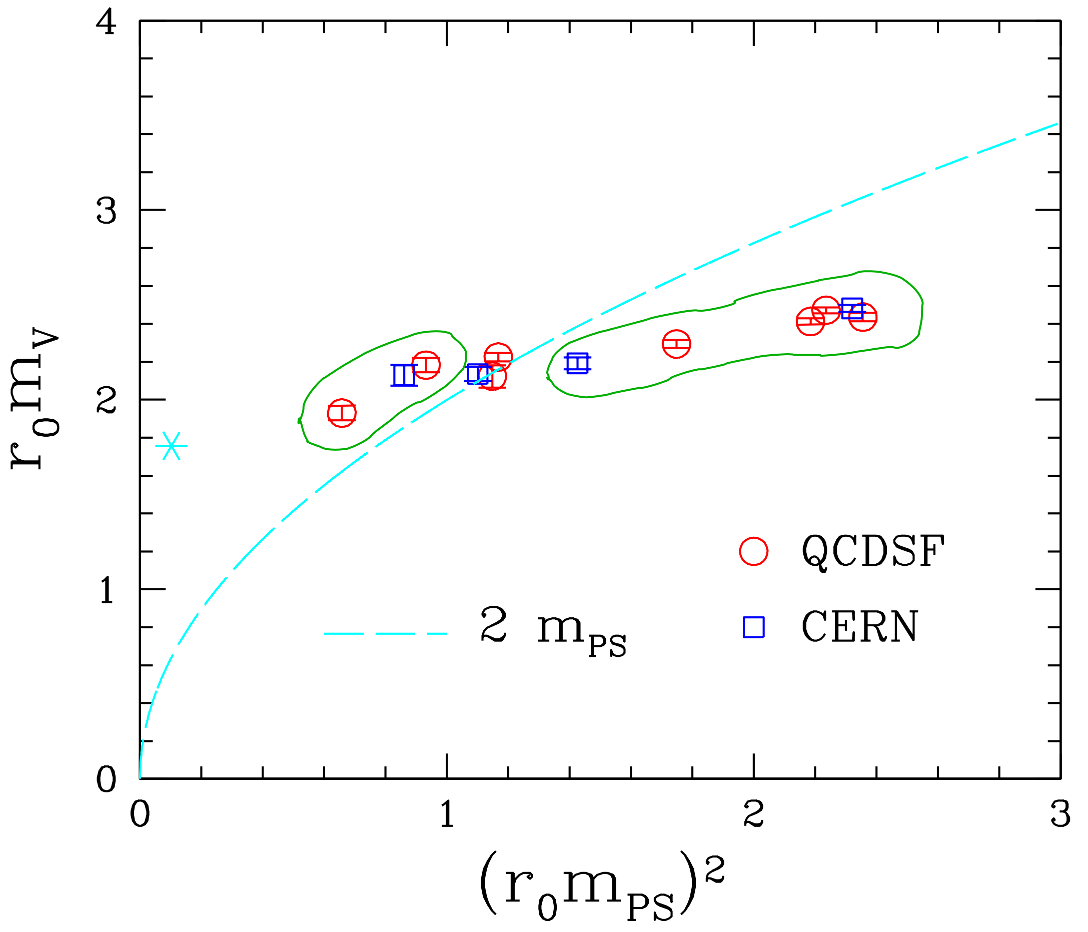


Z_A cancels , FS effects & leading log's largely cancel

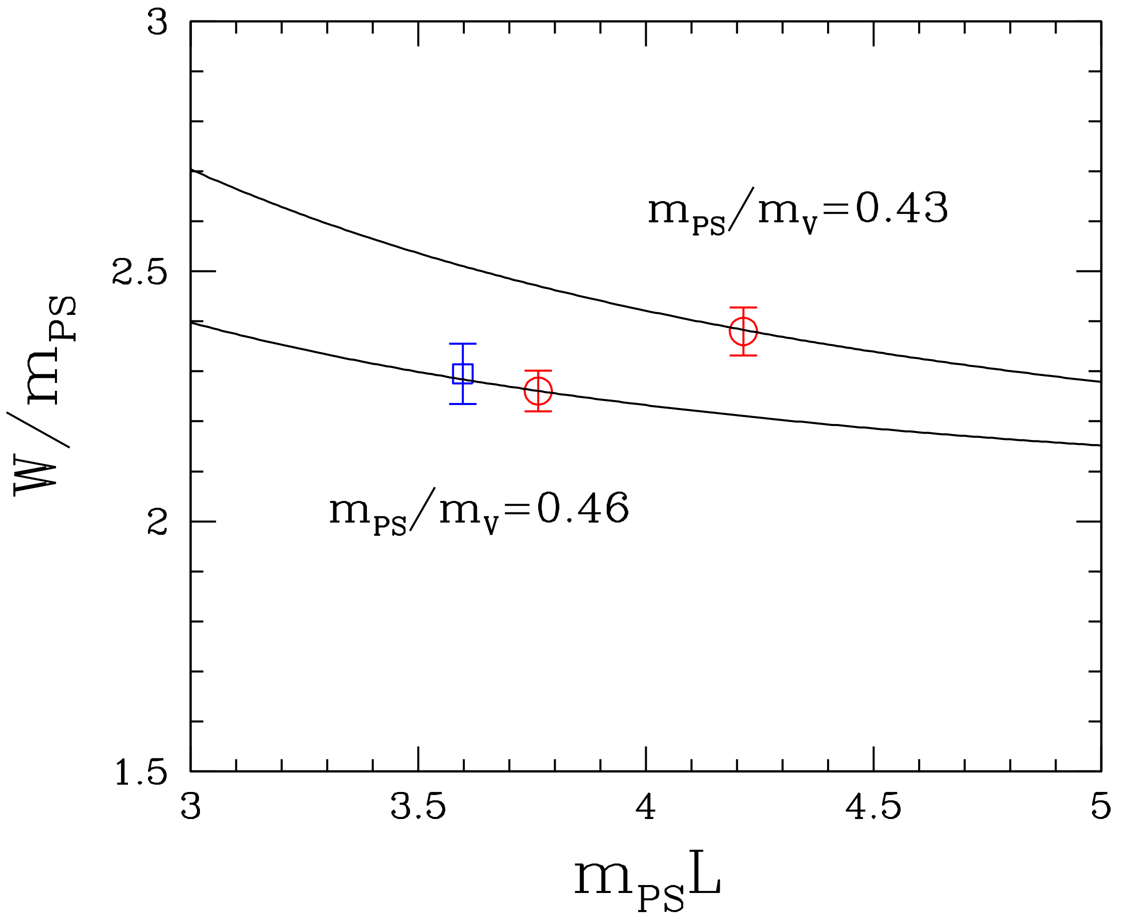
$r_0 = 0.45(1)$ fm

Rho Sector

Rho Mass



Raw data

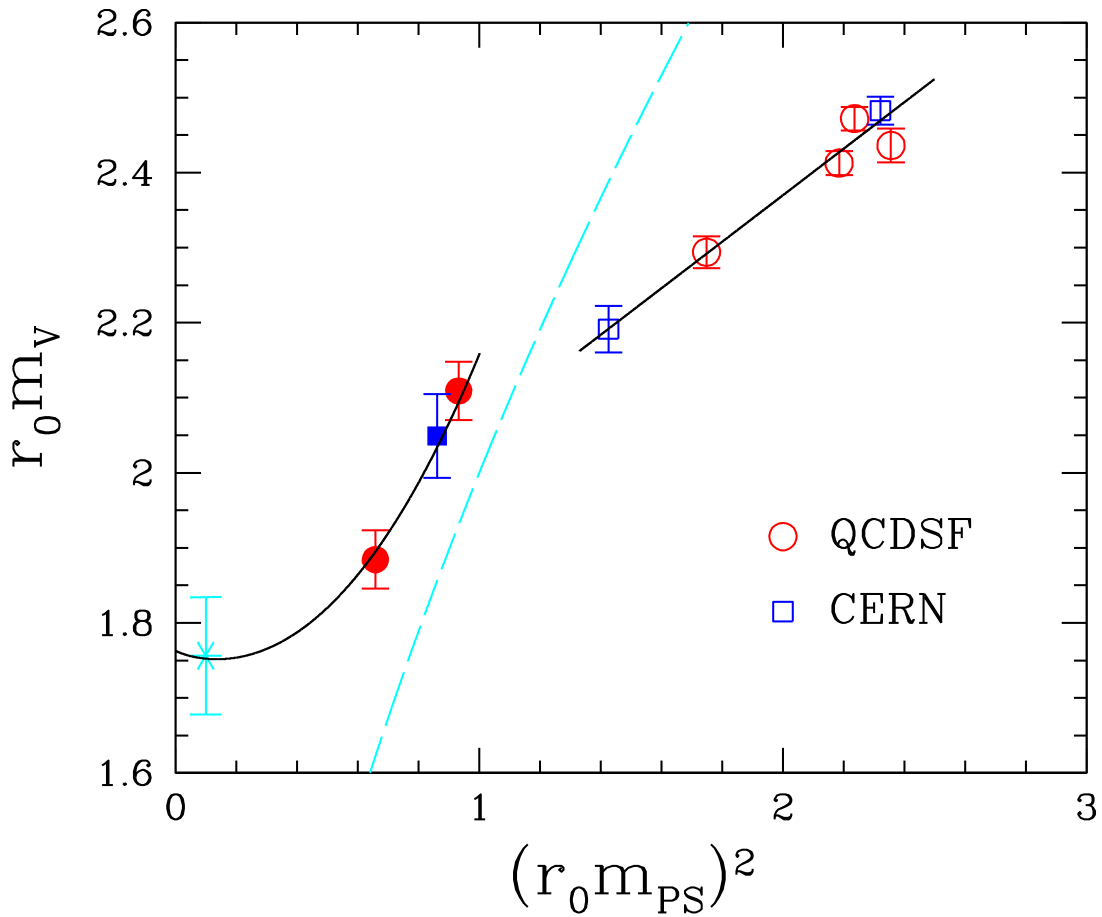


$$W = 2\sqrt{m_{PS}^2 + k^2}\,, \quad \delta_{11}(k) = \frac{(m_{PS}L)^2}{16\pi}\left[4 - \left(\frac{W}{m_{PS}}\right)^2\right] \mathrm{atan}\left(\frac{2\pi^2 q^2}{1 + 8.914 q^2 + \cdots}\right)$$

$$q = \frac{kL}{2\pi}$$

Lüscher

Physical Mass



Bruns & Meissner

Effective range formula: $\frac{k^3}{W}\cot\delta_{11} = \frac{4k_\rho^5}{m_\rho^2\Gamma_\rho}\left(1-\frac{k^2}{k_\rho^2}\right)\ ,\quad \Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k_\rho^3}{m_\rho^2}$ $\Longrightarrow$ m_ρ

Conclusions and Outlook

- Simulations at pion masses of $O(300)$ MeV with Wilson-type fermions feasible now
  - Improvement of algorithms
  - Increase of computing power

- Extrapolation to chiral limit and infinite volume greatly improved

  FS corrections surprisingly well described by ChPT

- First study of $\rho \longrightarrow \pi\pi$ decay on the lattice

- $f_\pi/g_A \Rightarrow r_0 = 0.45(1)$ fm – lower than usually assumed

  Lattices smaller than one has thought

- Future of Wilson-type fermions ?
  - $\mathcal{O}(20)$ times faster than DWFs
  - Symanzik improved gauge action
  - Stout links
  - $N_f = 2 + 1$