Quark mass determination from 2+1 flavor domain wall fermion simulations

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\( N_f = 2 + 1 \) Domain Wall Fermions ensembles

- \( N_f = 2 + 1 \) ensembles with Iwasaki gauge action part (publicly) available at http(s)://qcdlattices.bnl.gov generated on QCDDOC machines at Edinburgh and BNL
  * generated: \( 16^3 \times 32 \times 16 \), \( 24^3 \times 64 \times 16 \)
  * in production: \( 32^3 \times 64 \times 16 \) talk by Chulwoo Jung

- RHMC - algorithm talk by Norman Christ

  - \( 24^3 \times 64 \times 16 \) - ensemble
    * \( \beta = 2.13 \) (Iwasaki)
    * dynamical light quark mass: \( m_l \in \{0.005, 0.01, 0.02, 0.03\} \)
    * dynamical strange quark mass: \( m_s = 0.04 \)
    * lightest \( m_\pi \approx 330 \text{ MeV} \), \( m_l : m_{\text{strange}} = 1/5 \)
    * \( a^{-1} = 1.72(3) \text{ GeV} \) (from \( \Omega^- \))
    * \( a m_{\text{res}} \approx 0.0031 \): \( a m_x^{\text{phys}} = a m_x^{\text{bare}} + a m_{\text{res}} \)

- valence quark mass (partial quenching): \( m_{\text{val}} \in \{0.001, 0.005, 0.01, 0.02, 0.03, 0.04\} \)
  \[ \Rightarrow \text{lightest valence quark mass: } 1/10 m_{\text{strange}} \]
• use $\Omega^-$ baryon (sss) and Kaon (made from valence quarks)
• extrapolate to chiral limit (light dynamical quark mass, light valence quark in Kaon)
• strange quark mass from ratio $m_K^2 / m_{\Omega}^2$

$$a m_{\text{strange}}^{\text{phys}} = 0.0388(17)$$

• lattice scale from $\Omega^-$ mass at $a m_{\text{strange}}$:

$$a^{-1} = 1.722(27) \text{GeV}$$
extracting the light quark mass: Chiral Perturbation Theory

also talk by Meifeng Lin

• can we fit with $\chi$PT up to Kaon mass? Beyond??
• is LO+NLO enough?
• or do we need NNLO ??
  formulae from BijnenS et al. — add analytic terms (from symmetry considerations)

→ first we compare $SU(3) \times SU(3)$ vs. $SU(2) \times SU(2)$
  no strange quark mass in $SU(2)$:
    • unambiguous theoretical formulation — no worry about too heavy strange quark
    • extrapolation in strange quark mass, systematic error?

→ kaon mass and decay constant from

$$SU(2) \times SU(2) \text{ plus heavy strange}$$
SU(3) × SU(3)

- combined fit for
  \[ af_{xy} \]
  \[ \frac{(am_{xy})^2}{(am_{phys}^x + am_{phys}^y)/2} \]
  \[ am_{bare}^{\text{avg}} \leq 0.015 \]

- from \( m_\pi \):
  \[ (139.6 \text{ MeV}) \]
  \[ am_{ud}^{\text{phys}} = 0.001403(66) \]
  \[ \rightarrow m_{ud}/Z_m = 2.416(81)\text{MeV} \]
  \[ \rightarrow f_\pi = 126.9(3.1)\text{MeV} \]
  \[ (130.7 \text{ MeV}) \]
\[ SU(2) \times SU(2) \]

- combined fit for
  \[ a f_{xy} \]
  \[ \frac{(am_{xy})^2}{(am_{phys}^x + am_{phys}^y)/2} \]
  \[ am_{bare}^{avg} \leq 0.01 \]

- from \( m_{\pi} \):
  \( (139.6 \text{ MeV}) \)
  \[ am_{ud}^{phys} = 0.001403(63) \]
  \[ \rightarrow m_{ud}/Z_m = 2.417(77)\text{MeV} \]
  \[ \rightarrow f_{\pi} = 124.2(3.5)\text{ MeV} \]
  \( (130.7 \text{ MeV}) \)
• convert $B_0, f_0$, LECs from SU(3)-case to SU(2)
  (1-loop matching in Gasser, Leutwyler, 1985, 2-loop: Gasser et al. 2006)

• compare SU(2)-LECs at scale $m_\pi = 139$ MeV: $\bar{l}_{3,4}$

<table>
<thead>
<tr>
<th></th>
<th>$aB_0 \cdot Z_m$</th>
<th>$a f_0$</th>
<th>$\bar{l}_3$</th>
<th>$\bar{l}_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(2) $\times$ SU(2)</td>
<td>2.414(61)</td>
<td>0.0665(21)</td>
<td>3.13(33)</td>
<td>4.42(14)</td>
</tr>
<tr>
<td>SU(3) $\times$ SU(3)</td>
<td>2.453(75)</td>
<td>0.0662(17)</td>
<td>2.87(28)</td>
<td>4.09(05)</td>
</tr>
<tr>
<td>MILC ($N_f = 2 + 1$)</td>
<td></td>
<td>0.6(1.2)</td>
<td>3.8(5)</td>
<td></td>
</tr>
<tr>
<td>ETMC ($N_f = 2$)</td>
<td></td>
<td>3.62(12)</td>
<td>4.52(6)</td>
<td></td>
</tr>
<tr>
<td>CERN ($N_f = 2$)</td>
<td></td>
<td>3.0(5)</td>
<td>4.4(2)</td>
<td></td>
</tr>
</tbody>
</table>

• good agreement between SU(2) and SU(3)

• systematic error from fixed strange quark mass?
SU(2) × SU(2) plus heavy strange

• only use chiral symmetry properties of the up- and down-quarks (SU(2) × SU(2))

• treat strange quark as heavy: ideally \( m_{\text{strange}} \gg m_{\text{up,down}} \)

  * PDG: 95 ± 25 MeV \( \gg 2.5 \sim 5.5, \) MeV ?
  * our simulation, dynamical: \( 0.04 + a m_{\text{res}} \gg \{0.005, 0.01\} + a m_{\text{res}} \) ??

• we only have one dynamical strange quark mass (at the moment)
  using partially quenched as well for the moment

  future simulations optimally should include a 2nd set of ensembles
  using a different dynamical strange quark

• Sharpe, Zhang (1996), Booth (1995)
SU(2) × SU(2) plus heavy strange I: \( m_K^2 \)

- only use SU(2) × SU(2) for light (dynamical, valence) quarks
- treat \( a m_s \) as heavy

\[
m_K^2 = B_{0k}^{m_s} \left\{ 1 + \frac{d_1^{m_s}}{f_0^2} \chi_l + \frac{d_2^{m_s}}{f_0^2} \chi_x \right\}
\]

- \( f_0, B_0 \) from SU(2) × SU(2) fit, fixed, \( a m_l \leq 0.01 \)
- extrapolate to \( a m_{\text{phys}} \)
- use \( a m_s = (0.02), 0.03, 0.04 \)
• interpolate to $am_{\text{strange}}$
  
  $m_K = 512.1(1.3)\text{ MeV}$
  
  (493.7 MeV)

• or (again) extract $am_s$ from physical $m_K$:

  * $am_{\text{phys}}^{\text{strange}} = 0.0359(16)$
    
    compare with 0.0388(17) from $\Omega^-$

  $\Rightarrow$ systematic error

  (but no recalculation of $a^{-1}$ done)
SU(2) × SU(2) plus heavy strange II: \( f_K \)

- only use SU(2) × SU(2) for light (dynamical, valence) quarks
- treat \( a m_s \) as heavy

\[
f_K = f_0^{m_s} \left\{ 1 + \frac{c_1^{m_s}}{f_0^2} \chi_{ud} + \frac{c_2^{m_s}}{f_0^2} \chi_{x} - \frac{1}{(4\pi f_0)^2} \left[ \frac{\chi_x + \chi_{ud}}{2} \log \frac{\chi_x + \chi_{ud}}{2(a\Lambda)^2} + \frac{\chi_{ud} - 2\chi_x}{4} \log \frac{\chi_x}{(a\Lambda)^2} \right] \right\}
\]

- \( f_0, B_0 \) from SU(2) × SU(2) fit, fixed, \( a m_l \leq 0.01 \)
- extrapolate to \( a m_{ud}^{\text{phys}} \)
- use \( a m_s = (0.02), 0.03, 0.04 \)
• interpolate to $a m_{\text{strange}}$
  • $f_K = 150.0(3.6)$
    (159.8 MeV)
  • $f_K/f_\pi = 1.208(14)$
    (1.223)

• or (again) extract $a m_{\text{strange}}$ from physical $f_K$:
  * $a m_{\text{strange}}^{\text{phys}} = 0.060(10)$
    compare with 0.0388(17) from $\Omega^-$
  * systematic error
    (but no recalculation of $a^{-1}$ done)
• small slope in interpolation, not a good way
  to determine $a m_{\text{strange}}$
non-perturbative renormalization (Rome-Southampton)

- renormalization done at $16^3 \times 32 \times 16$ lattices (same gauge action, lattice spacing)
- match bare lattice operators to RI/MOM non-perturbatively
- perturbative matching to $\overline{\text{MS}}$ at 2 GeV
- **Domain Wall Fermions:**
  * control of chiral symmetry breaking
  * $O(a)$-improved
    $\rightarrow$ operator mixing reduced
    $\rightarrow$ (partially) conserved axial and vector currents
- publication in preparation by RBC- and UKQCD-Collaborations
- here we are interested in $Z_m = 1/Z_S$
• renormalized amp. vertex functions $\Lambda_i^{\text{ren}} = Z_i/Z_q \Lambda_i = 1, \ i \in \{S, P, V, A, T\}$

$$Z_{m}^{\text{RI}} = \frac{Z_q(p)}{Z_S(\Lambda_S)} \frac{Z_A(p)}{Z_q(\Lambda_A)} \frac{1}{Z_A} \text{ hadronic ME}$$

• four loop RG-running $Z_{m}^{\text{RGI}} = \frac{c(\alpha_s(\mu_0)/\pi)}{c(\alpha_s(\mu)/\pi)} Z_{m}(\mu)$

[Chetyrkin et al., 2000]

• three loop matching RI/MOM to $\overline{\text{MS}}$

$$Z_{m}^{\overline{\text{MS}}}(2 \text{ GeV}) = 1.575(28)(15)(83)$$

(error: statistical, $\Lambda_A \leftrightarrow \frac{1}{2}(\Lambda_A + \Lambda_V)$, linear vs. quadratic chiral extrap.)

E. E. Scholz (RBC/UKQCD) — Quark mass determination from 2+1 DWF
before I summarize: Disclaimer

- finite size effects
- continuum extrapolation
- interpolation in strange quark mass
- isospin-breaking, EM-splitting

need further investigation!
Finale: the quark masses

using the non-perturbative renormalization factor $Z_{m}^{\overline{MS}}(2 \text{ GeV}) = 1.58(09)$ (combined error):

- average light quark mass from pion mass in SU(2) × SU(2) $\chi$PT:
  
  $$m_{ud} = 3.82(25) \text{ MeV}$$

  systematic error estimate ??

- strange quark mass from $\Omega^{-}$ and Kaon mass:
  
  $$m_{\text{strange}} = 105.7(6.8) \text{ MeV} \quad m_{ud} : m_{\text{strange}} = 1 : 27.7(4)$$

  * obtained in SU(2) × SU(2) plus heavy strange

  $$m_{\text{strange}} = 97.6(6.2) \text{ MeV} \quad m_{ud} : m_{\text{strange}} = 1 : 25.6(3)$$

  (caveat: $a^{-1}$ not adjusted)

  * (rough) estimate for systematic error: 10%