Pseudo-scalar meson form factors
with maximally twisted Wilson fermions at $N_f = 2$

on behalf of
- The ETM collaboration has recently produced a large number of gauge configurations with two dynamical quarks at three lattice spacings and various lattice volumes adopting
  a) the tree-level Symanzik improved gauge action;
  b) twisted Wilson fermions tuned at maximal twist.

- The collaboration is presently computing a large number of phenomenologically relevant quantities based on 2pts and 3pts correlation functions for both mesons and baryons.

- The “3pts” program includes electromagnetic and weak semileptonic form factors of light and heavy-light mesons as well as of baryons.

- My aim is to present preliminary results concerning the calculation of pseudo-scalar meson form factors, in particular the vector and scalar form factors of the pion, the universal Isgur-Wise function, semileptonic $K_{l3}$ form factors, …

- we have completed the runs for 3 values of the sea quark mass @ $\beta = 3.9$ (a $\sim 0.09$ fm) and $L^3 T = 24^3 \times 48$
3pts correlators:

\[
C^K_\pi(t_x, t_y) = \sum_{\bar{\mathbf{x}}, \bar{\mathbf{y}}} \left\langle O_\pi(t_y, \bar{\mathbf{y}}) \hat{V}_\mu(t_x, \bar{\mathbf{x}}) O_K^\dagger(0) \right\rangle e^{-i\bar{p}_K \cdot \bar{\mathbf{x}} + i\bar{p}_\pi (\bar{\mathbf{x}} - \bar{\mathbf{y}})}
\]

\[
\frac{\sqrt{G_K G_\pi}}{4E_K E_\pi} \frac{\langle \pi | \hat{V}_\mu | K \rangle}{4 E_3 E_2} e^{-E_K t_x - E_\pi (t_y - t_x)}
\]

2pts correlators:

\[
C^K_\pi(t_x) = \sum_{\bar{\mathbf{x}}} \left\langle O_\pi(t_x, \bar{\mathbf{x}}) O_K^\dagger(0) \right\rangle e^{-i\bar{p}_K(t_x) \cdot \bar{\mathbf{x}}}
\]

\[
\frac{G_K}{2E_K} e^{-E_K(t_x)}
\]

* **local interpolating PS fields and current:** \( O_K = \bar{d} \gamma_5 s, \quad O_\pi = \bar{d} \gamma_5 u, \quad \hat{V}_\mu = Z_V \bar{s} \gamma_\mu u \)

* **twisted boundary conditions:** quark with momentum \( 2\pi \bar{\theta} / L \) (Bedaque, Roma-ToV, …)

\[
\sum_z D^\theta(x, z) S^\theta(z, 0) = \delta_{x,0}
\]  

rephasing: \( U_{\mu}^\theta(x) = e^{2\pi i \alpha \theta_\mu / L} U_\mu(x) \)

**K_{13} decay**

\[
\begin{align*}
\bar{p}_K &= \frac{2\pi}{L} (\bar{\theta}_3 - \bar{\theta}_1), \\
\bar{p}_\pi &= \frac{2\pi}{L} (\bar{\theta}_3 - \bar{\theta}_2), \\
\bar{q} &= \bar{p}_K - \bar{p}_\pi = \frac{2\pi}{L} (\bar{\theta}_2 - \bar{\theta}_1)
\end{align*}
\]
- to improve the precision (particularly at low quark masses) we decided to use the all-to-all propagator

\[ \sum_{z} D^\tilde{\theta}(x, z) S^\tilde{\theta}(z, y) = \delta_{x,y} \]

* stochastic sources: \( S^\tilde{\theta}(x, y) \rightarrow \frac{1}{N_s} \sum_{r=1}^{N_t} \Phi_r^\tilde{\theta}(x) \left[ \eta_r(y) \right] \)\(^\dagger\) (color, spin and flavor labels dropped)

\[ \sum_{z} D^\tilde{\theta}(x, z) \Phi_r^\tilde{\theta}(z) = \eta_r(x) \]

more precisely: \( \eta''(x) \Rightarrow \eta''_c(\bar{x}) \delta_{r,\text{source}} \) and one inversion for each spin on the same source

[“one-end” method: UKQCD (’06)]

- time slice randomly chosen for each gauge conf.

* 2pts correlators: \( C_{K(\pi)}^{K(\pi)}(t_x) \rightarrow \frac{1}{N_s} \sum_{r=1}^{N_t} \sum_{\bar{x}} \left\langle Tr \left\{ \gamma_5 \left[ \Phi_r^{\tilde{\theta}}(t_x, \bar{x}) \right] \gamma_5 \Phi_r^{0}(t_x, \bar{x}) \right\} \right\rangle \)

* 3pts correlators: \( C_{\mu}^{K\pi}(t_x, t_y) \rightarrow \frac{1}{N_s} \sum_{r=1}^{N_t} \sum_{\bar{x}} \left\langle Tr \left\{ \gamma_5 \left[ \Phi_r^{-\tilde{\theta}}(t_x, \bar{x}) \right] \gamma_\mu \Sigma_r^\tilde{\theta}(t_x, \bar{x}; t_y) \right\} \right\rangle \)

\[ \sum_{z} D^\tilde{\theta}(x, z) \Sigma_r^\tilde{\theta}(z; t_y) = \gamma_5 \Phi_r^{0}(x) \delta_{t_x, t_y} \] (generalized propagator)
Breit-frame:

\[ \vec{p}_K = -\vec{p}_\pi = \frac{\bar{q}}{2} \]

\[ \bar{\theta}_2 = -\bar{\theta}_1 \equiv \bar{\theta} \]

\[ \bar{\theta}_3 = 0 \]

\[ q^2 = \left( \sqrt{M_K^2 + \left( \frac{2\pi}{L} \theta \right)^2} - \sqrt{M_\pi^2 + \left( \frac{2\pi}{L} \theta \right)^2} \right)^2 - \left( \frac{2\pi}{L} 2\theta \right)^2 \]

for \( M_K = M_\pi \):

\[ q^2 = -\left( \frac{2\pi}{L} 2\theta \right)^2 \]

* Breit-frame: yes (the average of \( +\theta \) and \( -\theta \) corresponds to average \( K \rightarrow \pi \) and \( \pi \rightarrow K \) in the Breit frame)

* O(a)-improvement: remarkable improvement of the precision !!!

80 confs. only

\[ M_\pi \sim 300 \text{ MeV} \]

\[ aM_V = 0.44 \]
Analyses in progress:

1) \( m_1 = m_2 = m_{sp} = m_{sea} \)

2) \( m_1 = m_2 = m_{heavy} \)

3) \( m_1 = m_{strange} \) and \( m_2 = m_{sp} = m_{sea} \)

4) \( m_1 = m_{heavy} \) and \( m_2 = m_{sp} = m_{sea} \)

\[
\text{am}_{sea} = \{0.0040, 0.0064, 0.0085, 0.0100, 0.0150\}
\]

“light” quarks

\[
\text{am}_1, \text{am}_2 = \{0.0040, 0.0064, 0.0085, 0.0100, 0.0150, \\
0.022, 0.027, 0.032, \}
\]

“strange” quarks

\[
0.25, 0.32, 0.39, 0.46\}
\]

“heavy” quarks

- multiple mass solver [Jegerlehner (‘98)] on 240 confs. (1 out of 20 traj.) for each \( m_{sea} \)
* determination of $Z_V$

- from 2pts: 
  \[ Z_V = \frac{f_\pi M_\pi}{\sqrt{G_\pi}} \left[ \frac{\langle P^1_5 P^1_5 \rangle}{\langle A^1_0 P^1_5 \rangle} \right]_{t \to \infty} \]

  \[ f_\pi = 2m \sqrt{G_\pi} / M^2_\pi \quad \text{(WI in tmLQCD, } Z_m = 1/Z_P) \]

- from 3pts: 
  \[ Z_V \langle \pi(\bar{0}) | V_0^{(e.m.)} | \pi(\bar{0}) \rangle = 2M_\pi \quad F_\pi (q^2 = 0) = 2M_\pi \]

\[ m_{sp} = m_{sea} = m \]

see parallel talk by P. Dimopolous

statistical precision of 0.03%
* vector form factor of the pion: \( F_\pi(q^2) = \frac{Z_V}{2E_\pi} \langle \pi(\vec{0}) | V_0^{(e.m.)} | \pi(-\vec{0}) \rangle \)

- quality of the plateaux

- pole behavior O.K.

\[ F^{pole}(q^2) = \frac{1}{1 - \frac{\langle r^2 \rangle}{6} q^2} \]

lattice points exhibit a quite nice statistical precision
* overestimate of exp. points

* chiral behavior is a delicate issue

NLO: G&L ('84)

NNLO: Bijnens et al. ('98)

\[
\log(M_\pi^2) \text{ only!}
\]

\[
\langle r^2 \rangle (NLO) = \frac{2}{(4\pi F)^2} \left[ \log \left( \frac{\Lambda_6^2}{M_\pi^2} \right) - 1 \right]
\]

fix \( aF = 0.0534 \) from PLB '07

exp. \( \Rightarrow \bar{l}_6 = 14.4 (3) \)

\[
\langle r^2 \rangle (NNLO) = \langle r^2 \rangle (NLO) + c_1 M_\pi^2 + c_2 M_\pi^2 \log(M_\pi^2)
\]

ETMC + exp. \( \Rightarrow \bar{l}_6 = 17.2 (7) \)
* scalar form factor of the pion: \( F_{\pi}^{(s)}(q^2) = Z_p \langle \pi(\bar{\theta}) | \bar{u}u + \bar{d}d | \pi(-\bar{\theta}) \rangle \)

in tmLQCD (at maximal twist)

- pole behavior O.K.

\[
F_{\text{pole}}(q^2) = \frac{F_S(0)}{1 - \frac{\langle r^2 \rangle_S}{6} q^2}
\]

\[
\log(M_\pi^2) \text{ only!}
\]

\[
\langle r^2 \rangle_S(\text{NLO}) = \frac{12}{(4\pi F)^2} \left[ \log\left( \frac{\Lambda_4^2}{M_\pi^2} \right) - \frac{13}{12} \right]
\]

fix \( aF = 0.0534 \) and \( \bar{\ell}_4 = 4.52 \) from PLB '07

\[
\langle r^2 \rangle_S(\text{NNLO}) = \langle r^2 \rangle_S(\text{NLO}) + d_1 M_\pi^2 + d_2 M_\pi^2 \log(M_\pi^2)
\]
**Isgur-Wise function:**

\[
\xi(\omega) = \lim_{m_{\text{heavy}} \to \infty} F_{PS} \left( q^2 \right) = \frac{Z_v}{2E_{PS}} \left\langle PS(\bar{\theta}) \right| V_0 \left| PS(-\bar{\theta}) \right\rangle
\]

\[
\omega = 1 - \frac{q^2}{2M_{PS}}
\]

- pole behavior:
  
  \[
  \xi_{\text{pole}}(\omega) = \left( \frac{2}{1 + \omega} \right)^2 \rho_{\text{IW}}^2
  \]

- mild dependence on \( m_{\text{heavy}} \)
slope of IW function:

linear fit: $\rho_{iw}^2(u,d) = 0.77 \pm 0.28$

$N_f = 0$:

- $\rho_{iw}^2 = 0.83^{+15}_{-11-1}$ (UKQCD)
- $\rho_{iw}^2 = 1.24 \pm 0.26 \pm 0.26$ (MILC)
- $\rho_{iw}^2 = 0.89 \pm 0.17$ (Roma-ToV)
\[ \langle \pi | \hat{V}_\mu | K \rangle = (p_K + p_\pi) f_+ (q^2) + (p_K - p_\pi) f_- (q^2) \]

\[
\begin{align*}
fo(q^2) &= f_+(q^2) + f_- (q^2) q^2 / (M_K^2 - M_\pi^2) \\
fo(q^2) &= f_+(q^2) + f_- (q^2) q^2 / (M_K^2 - M_\pi^2)
\end{align*}
\]

very precise thanks to a 3pts double ratio

**K_{l3} decay**

see parallel talk by C. Tarantino on the strange sector

**heavy-light semileptonic decay**

see parallel talk by B. Blossier on the charm sector
Conclusions

* The ETM collaboration has started a program of intensive calculations of 3pts correlation functions using the ETMC unquenched gauge configurations, with the aim of addressing the phenomenology of both meson and baryon electroweak form factors.

* Thanks to algorithm improvements (mainly the use of a stochastic method to evaluate all-to-all propagators) it is possible to achieve a remarkable statistical precision in the determination of the form factors, in particular at the lowest pion mass of \( \sim 300 \text{ MeV} \).

* We have presented the preliminary results of our first runs, performed at a single lattice spacing (of \( \sim 0.09 \text{ fm} \)) and lattice volume \( (24^3 \times 48) \) for three values of the sea quark mass, concerning the vector and scalar form factors of the pion, the universal Isgur-Wise function, the weak form factors relevant in \( K_{13} \) and heavy-light semileptonic decays.

* As for the charge form factor of the pion, our lattice points overestimate the experimental data. Charge and scalar radii are expected to be the best candidates to see chiral enhancements. Indeed, chiral extrapolations guided by one- and two-loops ChPT seem to suggest the presence of relevant non-local terms. However, results at more values of the sea quark mass, as well as the investigation of volume effects and continuum extrapolation, are required in order to draw definite conclusions.