The Hadronic Spectrum of 2-colour QCD at Non-zero Chemical Potential

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Outline

1. Introduction and Motivation
   - Dense QCD and the Lattice
   - 2 Colour QCD

2. The Big Program
   - Overview
   - 2 Colour Spectroscopy

3. Results
   - Set up
   - Preliminary Results
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Introduction and Motivation

Dense QCD and the Lattice

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- A hard question
- Requires quantitative knowledge of the EOS of QCD
- It is known that at asymptotic densities, QCD enters a Colour superconducting CFL phase
- But between this and ordinary nuclear density very little is known
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Introduction and Motivation

Dense QCD and the Lattice

The Phase Diagram

Figure: A cartoon of the generally proposed phase diagram for QCD
Non-Zero $\mu$ and The Sign-Problem

Unfortunately within the current framework of lattice QCD, simulations with non-zero $\mu$ are beset by the “sign problem”, which makes general simulations impossible.

- In order to extract the non-perturbative physics, one is forced to either:
- Work in the regime where $\mu/T$ is small
  - Where techniques such as: analytic continuation from imaginary $\mu$ and multi-parameter re-weighting are valid.
- Or to work with toy models, and hope to infer some general properties.
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2 Colour QCD

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General Properties of 2cQCD

- Is the only dense matter system with long range interactions (gluons) that can be studied within the framework of Lattice QCD
- The quarks and anti-quarks live in equivalent representations of the colour group and can be related by an anti-unitary symmetry (the Pauli–Gürsey symmetry)
  - This leads to the chiral multiplet containing both $q \bar{q}$ mesons and $qq$ baryons
  - Also ensures that the fermion determinant is real
- Therefore with an even number of flavours it can be made to be positive definite and so have no sign problem
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It is therefore a useful laboratory to study gluodynamics at \( \mu \neq 0 \), especially the issue of deconfinement at high density.

The phase diagram has been studied with \( \chi PT \) (valid when \( m_\pi \ll m_\rho \)). The main results being:

- For \( \mu \geq \mu_0 = m_\pi / 2 \) a baryon charge density develops along with a superfluid condensate \( \langle qq \rangle \neq 0 \).
- For \( \mu \gtrsim \mu_0 \) the system is a dilute Bose-Einstein condensate of weekly interacting scalar qq baryons.
Introduction and Motivation

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The Phase Diagram

Figure: A diagram taken from Kogut, Toublan and Sinclair 2002, of a proposed phase diagram for 2 colour QCD
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The aim is to numerically investigate the $\mu$-axis of 2 colour QCD with 2 flavours of Wilson fermion.

Wilson not staggered because...

- Two-colour staggered lattice QCD has a different Goldstone spectrum to continuum 2cQCD (the pattern of global symmetry breaking is different)

- 4 flavours is uncomfortably close to the Banks-Zaks threshold

$$N_f = \frac{34N_c^3}{13N_c^2 - 3} \approx 5.6$$ (1)

where the second term of the $\beta$ function changes sign.
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The Big Program

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Some Results of The Collaboration

Evidence for three distinct regions have been found:

1. A vacuum phase, for $\mu < \mu_o \approx m_\pi/2$,
   - Baryon density remains zero

2. A confined, bosonic superfluid phase, for $\mu_o < \mu < \mu_d$,
   - Characterised by Bose–Einstein condensation of scalar diquarks.

3. A deconfined phase, for $\mu > \mu_d$,
   - Quarks and gluons are the dominant degrees of freedom
   - Evidence for a Fermi surface and non-zero binding energy, $k_F > E_F$.

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where $M(\mu)$ is the usual Wilson fermion matrix

$$M_{xy}(\mu) = \delta_{xy} - \kappa \sum_\nu \left[(1 - \gamma_\nu) e^{\mu \delta_{\nu 0}} U_\nu(x) \delta_{y,x+\hat{\nu}}ight.$$

$$\left. + (1 + \gamma_\nu) e^{-\mu \delta_{\nu 0}} U^\dagger_\nu(y) \delta_{y,x-\hat{\nu}} \right].$$  \hspace{1cm} (3)
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The Big Program

Overview

The Lattice Formulation

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- They lift the low-lying eigenmodes in the superfluid phase
- And enable the study of diquark condensation without any “partial quenching”.

In principle results should be extrapolated to the “physical” limit $J = \bar{J} = 0$
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The Lattice Formulation (continued)

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To perform the hadron spectroscopy, the Trinlat approach to All-to-All propagators is used.

- Enables physics beyond the flavour non-singlet spectrum to be studied
- Maximise the information extracted from a single gauge configuration

Trinlat method:

- Generate a single noise vector for each quark
- Dilute and invert the daughter vectors
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Initial spectroscopy is of the following particles:

- **Mesons**
  - Isovector pseudovector ("pion"), vector ("rho") and axial-vector
  - Isoscalar scalar

- **Diquarks**
  - Isovector axial-vector
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- The "Higgs and Goldstone" states
  - corresponding to the post-onset broken global $U(1)_V$ symmetry

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Estimates for the response of kaons can be made by measuring two propagators:

- One calculated in the usual way;
- The other from $\mu = 0$ inversions on the same $\mu \neq 0$ configuration;
  - i.e. this is partial quenching

Then stitch them together with the appropriate operators.

Note that the “strange” quark has the same mass as the normal one.
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Currently there is great interest in “Deeply bound Kaonic states” in the Nuclear physics community

- both from the theory side
- and experiment, KEK, LNF, GSI, BNL

One obvious area where such exotic states have big implications is in astrophysics

- Strange quark stars etc.

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Finite $\mu$ Predictions

- Predictions for the pion from Chiral perturbation theory

- An effective model with spin 1 particles, has predictions for the rho
Finite $\mu$ Predictions

- Predictions for the pion from Chiral perturbation theory

- An effective model with spin 1 particles, has predictions for the rho
The Hadronic Spectrum of 2-colour QCD at Non-zero Chemical Potential

Outline

1. Introduction and Motivation
   - Dense QCD and the Lattice
   - 2 Colour QCD

2. The Big Program
   - Overview
   - 2 Colour Spectroscopy

3. Results
   - Set up
   - Preliminary Results
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Results

Set up

Lattice Parameters

- All results were calculated on a $8^3 \times 16$ lattice
- $\beta = 1.7$, $\kappa = 0.178$
  - $a = 0.220(4)\,fm$, $m_\pi a = 0.79(1)$, $m_\pi/m_\rho = 0.80(1)$
- $j = 0.04$ unless otherwise stated ($j = J/\kappa$)
- The noise vectors were diluted in time, spin and flavour
- 100-400 configurations were typically used, (the larger $\mu$ values having the highest statistics)
- All results presented include only the connected parts
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Mesons (connected diagrams only, $j = 0.04$)
Meson Analysis

Before onset the pion and rho states remain (more or less) constant

Qualitatively follow the predictions from $\chi PT$ and the spin-1 effective model
- pion seems to increase too early
- rho a little late

States become increasingly noisy as $\mu$ increases

Isoscalar scalar and isovector axial vector mesons both decrease rapidly in the vacuum phase and remain light post onset
Diquarks (connected diagrams only, $j = 0.04$)
In the vacuum phase the diquarks scale as $\pm 2\mu$ as one would naively expect.

For $\mu > 0.25$ the heavier of each diquark pair become impossible to fit.

Post onset they appear to remain constant.
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Preliminary Results

Isoscalar Scalar Diquark Correlators

Graph showing the isoscalar scalar diquark correlators for different values of chemical potential ($\mu$) and spatial separation ($dt$). The data points are connected by lines of different colors and styles, indicating the variation with $\mu$. The graph plots the correlator values on a logarithmic scale against $dt$.
Preliminary Results

Higgs and Goldstone
Below onset it doesn’t make sense to distinguish between the higgs and goldstone

For $\mu > 0.4$ the states:

- Develop a mass splitting
- Scale differently with $J$

The mass of the goldstone appears to scale to zero in this phase (well at $\mu = 0.5$ at least)
The Hadronic Spectrum of 2-colour QCD at Non-zero Chemical Potential

Results

Preliminary Results

**Kaons** ($j = 0.04$)
As with the diquarks the heavier of each kaon pair rapidly becomes impossible to fit for $\mu \neq 0$

- Initially at least the kaons scale as $\pm \mu$
- In the dense phase the state is certainly lighter than that in vacuum ($\mu = 0$)
  - So would be bound if one was created in a blob of dense su(2) matter...
- Interestingly as $\mu$ increases post onset the kaonic state becomes heavier
A wide selection of hadronic states have been simulated and their response to $\mu$ investigated

Outlook

- More statistics
- Analysis of the disconnected terms
- Investigation of various smearing techniques
- A push to higher $\mu$ to look for signs of deconfinement

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