Semileptonic decays of heavy-light pseudoscalar mesons

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in the approximation of massless leptons (not $\tau$), the differential decay rate for the process $B \rightarrow D\ell\nu$ is

$$\frac{d\Gamma(B \rightarrow D\ell\nu)}{d\omega} = (\text{kin. fact.}) \times
\times |V_{cb}|^2 (\omega^2 - 1)^{\frac{3}{2}} \left[ G^{B \rightarrow D}(\omega) \right]^2$$

$$1 \leq \omega = \frac{p_B \cdot p_D}{M_B M_D} = v_B \cdot v_D \leq 1.6$$

The form factor $G^{B \rightarrow D}(\omega)$ is the non-perturbative input required to extract $V_{cb}$ from this channel.
the Fermilab group devised the "double ratio" technique to get $G^{B\to D}(\omega = 1)$ with high accuracy

[Hashimoto et al, Phys Rev D61 (2000)]
[Okamoto et al, Nucl Phys Proc Suppl 140 (2005)]
[talk by J. Laiho]

the factor $(\omega^2 - 1)^{3/2}$ forbids the direct experimental measurement of the decay rate at zero recoil:

- extrapolations
- the final error on $|V_{cb}|$ is a region in the plane $\langle G^{B\to D}(\omega = 1) \rangle - \langle \rho^2 \rangle$
the semileptonic decay $B \rightarrow D\ell\nu$ is mediated by the vector part of the heavy-heavy flavour changing vector current

$$\langle M_f | V^\mu | M_i \rangle = (v_i + v_f)^\mu \ h_+^{i \rightarrow f} + (v_i - v_f)^\mu \ h_-^{i \rightarrow f}$$

Time reversal and hermiticity imply that $h_+^{i \rightarrow f}$ and $h_-^{i \rightarrow f}$ are real. Furthermore

$$h_+(w, M_i, M_f) = + h_+(w, M_f, M_i)$$
$$h_-(w, M_i, M_f) = - h_-(w, M_f, M_i)$$

$$\epsilon_+ = \frac{1}{M_f} + \frac{1}{M_i}$$
$$\epsilon_- = \frac{1}{M_f} - \frac{1}{M_i}$$

$$h_+(w, \epsilon_+, \epsilon_-) = h_+(w, 0, 0) + \epsilon_+ \frac{\partial h_+(w, 0, 0)}{\partial \epsilon_+} + \frac{\epsilon_+^2}{2} \frac{\partial^2 h_+(w, 0, 0)}{\partial \epsilon_+^2} + \frac{\epsilon_-^2}{2} \frac{\partial^2 h_+(w, 0, 0)}{\partial \epsilon_-^2} + \ldots$$

$$h_-(w, \epsilon_+, \epsilon_-) = \epsilon_- \frac{\partial h_-(w, 0, 0)}{\partial \epsilon_-} + \frac{\epsilon_- \epsilon_+}{2} \frac{\partial^2 h_-(w, 0, 0)}{\partial \epsilon_- \partial \epsilon_+} + \ldots$$
the elastic case, $M_i = M_f$ or $\varepsilon_- = 0$, is constrained by vector symmetry

\[
\begin{align*}
    h_+(w = 1, \varepsilon_+, 0) &= 1 \\
    h_(w, \varepsilon_+, 0) &= 0 \\
    h_+(w = 1, 0, 0) &= 1 \\
    \frac{\partial^n h_+(w = 1, 0, 0)}{\partial \varepsilon_+^n} &= 0
\end{align*}
\]

it follows the Luke's theorem:

\[
h_+(w = 1, \varepsilon_+, \varepsilon_-) = 1 + \frac{\varepsilon_-^2}{2} \frac{\partial^2 h_+(w = 1, 0, 0)}{\partial \varepsilon_-^2} + \ldots
\]

furthermore, by usign HQET $h_+(w, 0, 0) = \xi(w)$, the Isgur-Wise function.

the form factor entering in the decay rate is given by

\[
G^{i\rightarrow f}(w) = h_+^{i\rightarrow f}(w) - \frac{M_f - M_i}{M_f + M_i} h_-^{i\rightarrow f}(w)
\]
form factors (lattice)

on the lattice we measured the following correlation functions

\[ \mathcal{F}^\mu_{i \rightarrow f}(x_0; \mathbf{p}_i, \mathbf{p}_f) = \frac{a^3}{2} \sum_x \langle O_{li} \, \mathcal{V}_{ii}^\mu(x) \, O_{fl}' \rangle = \]

\[ \mathcal{F}^\mu_{i \rightarrow i}(x_0; \mathbf{p}_i, \mathbf{p}_i) = \frac{a^3}{2} \sum_x \langle O_{li} \, \mathcal{V}_{ii}^\mu(x) \, O_{il}' \rangle = \]

\[ f_f^A(x_0, \mathbf{p}_f) = - \sum_x \langle O_{lf} \, A_{fl}^0(x) \rangle = \]

\[ \psi_{i,f}(x + \hat{1}L) = e^{i\theta_{i,f}} \psi_{i,f}(x), \quad p_1 = \frac{\theta_{i,f}}{L} + \frac{2\pi k_1}{L}, \quad k_1 \in \mathbb{N} \]
form factors (lattice)

the matrix elements are extracted from the following single ratios

\[
\langle V^i \rangle_{i \to f}^{D1} \equiv \langle M_f | V^i | M_i \rangle_{D1} \equiv 2\sqrt{M_i E_f} \frac{F_{i \to f}^\mu(T/2; 0, p_f)}{\sqrt{F_{i \to i}^0(T/2; 0, 0) F_{f \to f}^0(T/2; p_f, p_f)}}
\]

\[
\langle V^i \rangle_{i \to f}^{D2} \equiv \langle M_f | V^i | M_i \rangle_{D2} \equiv 2\sqrt{M_i E_f f_A(T/2, 0)} \frac{F_{i \to f}^\mu(T/2; 0, p_f)}{\sqrt{F_{i \to i}^0(T/2; 0, 0) F_{f \to f}^0(T/2; 0, 0)}}
\]

the form factors are defined as linear combinations of the matrix elements \( r = \frac{M_f}{M_i} \)

\[
h_{i \to f}^+(w) = \frac{\langle V^0 \rangle_{i \to f}}{2M_i \sqrt{r}} \left\{ 1 + \frac{\sqrt{w^2 - 1}}{w + 1} \frac{\langle V^1 \rangle_{i \to f}}{\langle V^0 \rangle_{i \to f}} \right\}
\]

\[
h_{i \to f}^-(w) = \frac{\langle V^0 \rangle_{i \to f}}{2M_i \sqrt{r}} \left\{ 1 + \frac{w + 1}{\sqrt{w^2 - 1}} \frac{\langle V^1 \rangle_{i \to f}}{\langle V^0 \rangle_{i \to f}} \right\}
\]

\[
G_{i \to f}^+(w) = \frac{2r}{1 + r} \frac{\langle V^0 \rangle_{i \to f}}{2M_i \sqrt{r}} \left\{ 1 + \frac{wr - 1}{r \sqrt{w^2 - 1}} \frac{\langle V^1 \rangle_{i \to f}}{\langle V^0 \rangle_{i \to f}} \right\}
\]

also \( w \) is expressed in terms of 3-point correlation functions

\[
x_f = \frac{F_{f \to f}^1(T/2; 0, p_f)}{F_{f \to f}^0(T/2; 0, p_f)} = \frac{\langle M_f | V^1 | M_f \rangle}{\langle M_f | V^0 | M_f \rangle} = \frac{\sqrt{w^2 - 1}}{w + 1}
\]
the step scaling method

the starting point of step scaling method calculation is

\[
F^{i \to f}(w; L_2) = F^{i \to f}(w; L_0) \sigma^{i \to f}(w; L_0, L_1) \sigma^{i \to f}(w; L_1, L_2)
\]

finite volume effects, measured by the step scaling functions, are insensitive to the high energy scale (parent quark mass)

\[
\sigma^{i \to f}(w; L_0, L_1) \equiv \frac{F^{i \to f}(w; L_1)}{F^{i \to f}(w; L_0)} = \frac{F^{(0) \to f}(w; L_1)}{F^{(0) \to f}(w; L_0)} \left[ 1 + \frac{F^{(1) \to f}(w; L_1)}{m_i} + \ldots \right] = \frac{F^{(0) \to f}(w; L_1)}{F^{(0) \to f}(w; L_0)} \left[ 1 + \frac{F^{(1) \to f}(w; L_1) - F^{(1) \to f}(w; L_0)}{m_i} + \ldots \right] = \sigma^{(0) \to f}(w; L_0, L_1) \left[ 1 + \frac{\sigma^{(1) \to f}(w; L_0, L_1)}{m_i} + \ldots \right]
\]
the step scaling method

the starting point of step scaling method calculation is

\[ F^{i \rightarrow i}(w; L_2) = F^{i \rightarrow i}(w; L_0) \sigma^{i \rightarrow i}(w; L_0, L_1) \sigma^{i \rightarrow i}(w; L_1, L_2) \]

finite volume effects, measured by the step scaling functions, are insensitive to the high energy scale (parent quark mass)

\[ \sigma^{i \rightarrow i}(w; L_0, L_1) \equiv \frac{F^{i \rightarrow i}(w; L_1)}{F^{i \rightarrow i}(w; L_0)} = \frac{F^{(0)}(w; L_1)}{F^{(0)}(w; L_0)} \left[ 1 + \frac{F^{(1)}(w; L_1)}{m_i} + \ldots \right] \]

\[ = \frac{F^{(0)}(w; L_1)}{F^{(0)}(w; L_0)} \left[ 1 + \frac{F^{(1)}(w; L_1) - F^{(1)}(w; L_0)}{m_i} + \ldots \right] \]

\[ \equiv \sigma^{(0)}(w; L_0, L_1) \left[ 1 + \frac{\sigma^{(1)}(w; L_0, L_1)}{m_i} + \ldots \right] \]
<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$T \times L^3$</th>
<th>$N_{\text{cnfg}}$</th>
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<tbody>
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<td>$L_0 A$</td>
<td>7.300</td>
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<tr>
<td>$L_0 B$</td>
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<td>$40 \times 20^3$</td>
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<td>$L_0 C$</td>
<td>6.963</td>
<td>$32 \times 16^3$</td>
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<td>$L_1 a$</td>
<td>6.737</td>
<td>$24 \times 12^3$</td>
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<td>$L_1 b$</td>
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<td>$16 \times 8^3$</td>
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<td>$L_2 b$</td>
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<td>$L_2 B$</td>
<td>5.960</td>
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finite volume results: $G^{B \rightarrow D}(w)$
finite volume results: $h^{B \rightarrow D}(w)$

![Graph of $h^{B \rightarrow D}(w)$]

The graphs show the finite volume results for the transition $h^{B \rightarrow D}(w)$, with plots for different masses $m_i$. The graphs likely represent the ratio of some quantity as a function of $w$, normalized by $\sigma_{+}^{L_{1}L_{2}}$. The data points are indicated with error bars, and the graphs are labeled with the masses $m_i = 0.138$, $m_i = 0.098$, and $m_i = 0.055$.
finite volume results: $h^{B \rightarrow D}(w)$

<table>
<thead>
<tr>
<th>$(a/r_0)^2$</th>
<th>$w=1.000$</th>
<th>$w=1.030$</th>
<th>$w=1.050$</th>
<th>$w=1.100$</th>
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<td></td>
<td>-0.16</td>
<td>-0.14</td>
<td>-0.12</td>
<td>-0.10</td>
<td>-0.08</td>
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<td>0.002</td>
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<tr>
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<td>0.006</td>
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<td>-0.12</td>
<td>-0.10</td>
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$m_l = 0.138$
$m_l = 0.098$
$m_l = 0.055$

$\sigma_{i \rightarrow f}(L_0,L_1)$
$\sigma_{i \rightarrow f}(L_0,L_2)$
finite volume results: consistency checks

\[ G^B \rightarrow D(L_0) \]
\[ h^B \rightarrow D(L_0) \]
\[ G^B \rightarrow D(L_2) \]
\[ h^B \rightarrow D(L_2) \]

\[ h^D \rightarrow D \] \( L (\text{fm}) \)
\[ h^D \rightarrow D \] \( L (\text{fm}) \)
physical results: $V_{cb}$

as an indication, we get: $V_{cb}(\omega = 1.2) = 3.84(9)(42) \times 10^{-4}$
### Physical Results: Comparison

<table>
<thead>
<tr>
<th>$w$</th>
<th>$G^{B \to D}(w)$</th>
<th>$N_f$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.026(17)</td>
<td>0</td>
<td>this work</td>
</tr>
<tr>
<td>1.03</td>
<td>1.001(19)</td>
<td>0</td>
<td>this work</td>
</tr>
<tr>
<td>1.05</td>
<td>0.987(15)</td>
<td>0</td>
<td>this work</td>
</tr>
<tr>
<td>1.10</td>
<td>0.943(11)</td>
<td>0</td>
<td>this work</td>
</tr>
<tr>
<td>1.20</td>
<td>0.853(21)</td>
<td>0</td>
<td>this work</td>
</tr>
<tr>
<td>1.00</td>
<td>1.058(20)</td>
<td>0</td>
<td>Hashimoto et al</td>
</tr>
<tr>
<td>1.00</td>
<td>1.07(24)</td>
<td>2+1</td>
<td>Okamoto et al</td>
</tr>
</tbody>
</table>
physical results: $\xi(w)$

$$m_{l} = m_{b} \quad m_{l} = 0.9 m_{b} \quad m_{l} = 0.6 m_{b} \quad m_{l} = 0.3 m_{b} \quad m_{l} = m_{c}$$

$$m_{f} = m_{b} \quad m_{f} = 0.6 m_{b} \quad m_{f} = 0.3 m_{b} \quad m_{f} = m_{c}$$

corrections to the static limits on $h^{B \rightarrow f}_{\pm}$

- are of the order of 2% at the charm mass
- completely negligible at $m_{b}/2$ (below 1%)
physical results: $\xi(w)$

as predicted by the Luke's theorem:

$$h_+(w = 1, \varepsilon_+, \varepsilon_-) = 1 + \frac{\varepsilon_-^2}{2} \frac{\partial^2 h_+(w = 1, 0, 0)}{\partial \varepsilon_-^2} + \ldots$$

from a linear fit we get the slope of the Isgur-Wise function: $\rho^2 = 0.92(15)$ (or $\rho^2 = 0.89(17)$ from a quadratic fit).
we have

- calculated both the form factors that parametrize pseudoscalar-pseudoscalar semileptonic transitions ($V_{cb}$ also from $B \rightarrow D \tau \nu_\tau$)

- data in the range $1 \leq w \leq 1.2$ where experimental data are directly available

- checked numerical the onset of HQET on these observables ($\xi(w)$)

- ok, still quenched but...
  
  - the step scaling method works impressively well on these observables
  
  - the calculation can be repeated with $N_f > 0$