$B-B$ Mixing with Domain Wall Fermions

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work within UKQCD and RBC collaborations

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Kanazawa: Taku Izubuchi
Outline

1. Motivation
2. The Static Approximation on the Lattice
3. Numerical Results
4. Summary
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1 Motivation

2 The Static Approximation on the Lattice

3 Numerical Results

4 Summary
Meson Mixing in the Standard Model

- box diagram with top quark dominant contribution

\[ \Delta m_q = -\frac{1}{6\pi^2} \left( G_F^2 m_W^2 \eta_B S_0 \right) m_{B_q} B_{B_q} f_{B_q}^2 (V_{tq}^* V_{tb})^2 \]

- non-perturbative part: \( B_{B_q} f_{B_q}^2 \)
- to extract CKM matrix elements:

\[ \frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2, \quad \text{with} \quad \xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} \]
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Constrains on the Unitarity Triangle

- summary of constraints on “unitarity triangle”
- circles from $B$ and $B_s$ mixing
- uncertainty totally dominated by theory error on matrix elements

$$\frac{|V_{td}|}{|V_{ts}|} = 0.2060 \pm 0.0007\,(\text{exp}) + 0.0081 - 0.0060\,(\text{theo})$$

[CDF, 2006]

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- limit of infinite $b$-quark mass
- lattice action: $S_h = a^4 \sum_x \bar{\psi}_h(x) D_0 \psi_h(x)$, $D_0$ covariant derivative in time direction

static propagator:

$$G_h(x, y) = \theta(x_0 - y_0) \delta(\vec{x} - \vec{y}) \mathcal{P}(y, x)^\dagger P_+,$$

$$\mathcal{P}(x, x + n\hat{\mu}) = \prod_{i=0}^{n-1} U_\mu(x + i\hat{\mu})$$

$$P_+ = \frac{1}{2}(1 + \gamma_0)$$

- divergence in self-energy of the static quark
- introduce counterterm: $(D_0 + \delta m) \tilde{G}_h(x, y) = \delta(x - y) P_+$
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  \[ G_h(x, y) = \theta(x_0 - y_0)\delta(x - y) P(y, x)^\dagger P_+ , \]

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  \[ P(x, x + n\hat{\mu}) = \prod_{i=0}^{n-1} U_\mu(x + i\hat{\mu}) \]

  \[ \mathcal{P}(y, x) = P(y, x)^\dagger P_+ \]

- Divergence in self-energy of the static quark

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Smeared Gauge Actions

- lattice action can be changed without changing continuum limit
  - only way here: smearing of gauge field
  - two possible choices:

  APE smearing:
  - replace link by sum of staples
  - $SU(3)$ projection

  Hypercubic blocking (HYP):
  - 3 steps of APE smearing
  - restricted to the hypercube around the original link
  - [Hasenfratz & Knechtli, 2001]

  3 parameters, we choose
  \[ (\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0, 0.5) \]
  - [ALPHA, Della Morte et al., 2004]
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Comparing Gauge Smearings

- heavy-light two-point correlation function with different smearings

- Eichten-Hill: no smearing
- APE smearing with $\alpha = 1.0$
- HYP1: $(\alpha_1, \alpha_2, \alpha_3) = (0.75, 0.6, 0.3)$
- HYP2: $(\alpha_1, \alpha_2, \alpha_3) = (1.0, 1.0, 0.5)$

- confirmation of ALPHA results, independent of light quark action
- test of our implementation of static action
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Domain Wall Fermions

- five dimensional formulation which has approximate chiral symmetry
- left-handed modes are bound to 4-D brane at $s = 0$, right-handed modes at $s = L_s$, exponentially suppressed overlap
- measure of chiral symmetry breaking: violation of 5-D Ward Identity:

$$m_{\text{res}} = \lim_{m \to 0} \frac{\langle \sum \bar{x} J_5 q(x) \pi(0) \rangle}{\langle \sum \bar{x} J_5 (x) \pi(0) \rangle}$$

- here: $m_{\text{res}} = 0.00308(4) \approx 5 \text{ MeV}$
- off-shell $O(a)$-improved, well suited for NPR in RI-MOM scheme
- renormalisation simplified by reduced operator mixing
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Renormalisation

- consider parity conserving operator $V^\mu V_\mu + A^\mu A_\mu$
- static approximation leads to mixing

$$O_{VV+AA}^{\text{ren}} = Z_{VA} O_{VV+AA} + Z_{SP} O_{SS+PP}$$

- perturbative results for APE and HYP smearing

[Dumitrescu, Loktik, Izubuchi, 2006, talk by Thomas a few minutes ago]

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- non-perturbative renormalisation à la Rome-Southampton

→ work in progress
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- consider parity conserving operator $V^\mu V_\mu + A^\mu A_\mu$
- static approximation leads to mixing

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[Dumitrescu, Loktik, Izubuchi, 2006, talk by Thomas a few minutes ago]

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Outline

1. Motivation
2. The Static Approximation on the Lattice
3. Numerical Results
4. Summary
Gauge Field Ensembles

- $2 + 1$ flavour Domain Wall fermions, $16^3 \times 32 \times 16$ lattices
- Iwasaki gauge action, $\beta = 2.13$, $a^{-1} = 1.62(4)$ GeV

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- construct ratio $\Phi = \frac{C_{WL}(t)}{\sqrt{C_{WW}(t) e^{-m^* t L^3}}}$
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APE:  
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APE: directly compute full matrix element $M = \frac{8}{3} m_B^2 f_B^2 B_B$ from ratio $\frac{\langle B^B | O_{LL} | B^B \rangle e^{m^*_B t_1/2}}{\sqrt{C^{BB}(t,t_1) C^{BB}(t,0)}}$ with box source of size $8^3$
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APE: \[ f_{B_s} \sqrt{B_{B_s}} = 262(14) \text{ MeV} \]
\[ f_B \sqrt{B_B} = 237(13) \text{ MeV} \]

HYP: \[ B_{B_s} = 0.791(16) \]
\[ B_B = 0.738(40) \]

\[ f_{B_s} \sqrt{B_{B_s}} = 273(14) \text{ MeV} \]
\[ f_B \sqrt{B_B} = 232(29) \text{ MeV} \]

\[ \xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_B \sqrt{B_B}} \]
\[ \xi^{\text{APE}} = 1.11(7), \quad \xi^{\text{HYP}} = 1.13(7) \]

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Outline

1. Motivation
2. The Static Approximation on the Lattice
3. Numerical Results
4. Summary
static approximation well defined limit of QCD, used as a reference point
- numerical simulations of matrix elements of static-light mesons
- preliminary results for decay constants and bag parameters for $B$ and $B_s$ meson
- linear chiral extrapolation
static approximation well defined limit of QCD, used as a reference point

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non-perturbative renormalisation

more and lighter data points, $\rightarrow 24^3 \times 64 \times 16$ (now)

finer lattice spacing, $\rightarrow 32^3 \times 64 \times 16$ (soon)

extent $b$-physics program:
- other 4-fermion operators (SUSY)
- semi-leptonic form factors
- $B$ meson lifetimes
- $\lambda_b$ baryons
To Do List

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Plans for NPR

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Jan Wennekers (Edinburgh)
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