Moving NRQCD and

\[ B \rightarrow K^*\gamma \]

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with
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 Flavor physics is “LHC-era physics.” We need to determine non-SM couplings as well as non-SM masses.

 Role for LQCD calculations well-known to this audience

 Need to study rare decays, e.g. $b \rightarrow s$, for possible new sources of FCNC

 LQCD calculation of form factors here are highly desired

 More challenges than tree-decays (clean phenomenology) and $B$ mixing (no momentum lost, stable final state, smaller contaminations/corrections)
### Full set of form factors

<table>
<thead>
<tr>
<th>Matrix element</th>
<th>Form factor</th>
<th>Relevant decay(s)</th>
</tr>
</thead>
</table>
| \( \langle P | \bar{q} \gamma^\mu b | B \rangle \) | \( f_+, f_0 \) | \( \{ \)
| \( \langle P | \bar{q} \sigma^{\mu\nu} q_\nu b | B \rangle \) | \( f_T \) | \( B \rightarrow K \ell^+ \ell^- \)
| \( \langle V | \bar{q} \gamma^\mu b | B \rangle \) | \( V \) | \( \{ \)
| \( \langle V | \bar{q} \gamma^\mu \gamma^5 b | B \rangle \) | \( A_0, A_1, A_2 \) | \( \{ B \rightarrow (\rho/\omega) \ell \nu \)
| \( \langle V | \bar{q} \sigma^{\mu\nu} q_\nu b | B \rangle \) | \( T_1 \) | \( \{ B \rightarrow K^* \ell^+ \ell^- \)
| \( \langle V | \bar{q} \sigma^{\mu\nu} \gamma^5 q_\nu b | B \rangle \) | \( T_2, T_3 \) | \( \{ B \rightarrow K^* \gamma \)
| \( \langle P | \bar{q} \sigma^{\mu\nu} \gamma^5 \gamma^5 q_\nu b | B \rangle \) | \( \langle P | \bar{q} \gamma^{\mu\nu} b | B \rangle \) | \( \{ B \rightarrow K^* \ell^+ \ell^- \)
|
Pertinent operator (in SM) (neglecting long distance contributions)

\[
\langle K^*(k, e_\lambda) | \bar{s} \sigma_{\mu\nu} \frac{(1 + \gamma_5)}{2} b | B(p) \rangle = 2 \epsilon_{\mu\alpha\beta\gamma} e_\lambda^{*\alpha} p^\beta k^\gamma (T_1(q^2)
\]

\[
+ i \left[ e_\lambda^{*\mu} (m_B^2 - m_{K^*}^2) - (e_\lambda \cdot q)(p + k)_\mu \right] T_2(q^2)
\]

\[
+ i (e_\lambda \cdot q) \left[ q^\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (p + k)_\mu \right] T_3(q^2)
\]
Our strategy

- mNRQCD for heavy quark
  - EFT permitting calculations directly at $m_b$
  - Larger range of $q^2$ than non-moving NRQCD

- Improved staggered action for light quark
  - MILC lattices, large set of sea masses in chiral regime
  - Issues: chiral extrapolation and $K^* \rightarrow K \pi$ threshold

- Automated perturbation theory
  - 1-loop operator matching
  - Automatic differentiation
Moving HQET/NRQCD

- History: Mandula & Ogilvie, Hashimoto & Mastufuru, Sloan, Davies-Dougall-Foley-Lepage-Wong
- Nonzero recoil: $B \to D$ form factors (Isgur-Wise)
- Lower $q^2$: $B \to \pi$ form factors
- Lowest $q^2$: $B \to K^* \gamma (q^2 = 0)$
mNRQCD: comments

- Possible to extend kinematically accessible range with $v > 0$
- As with NRQCD, it’s an EFT valid for $b$ quarks on existing and near-future lattice spacings
- “Cannot take the continuum limit.” No problem! Go to the renormalized trajectory using Symanzik improvement and perturbation theory
- Relies on HQET power counting to match operators
Limitation of HQET
Limitation of HQET

Figure of merit \( \frac{u \cdot p_f}{m_B} = \frac{p_B \cdot p_f}{m_B^2} \)

- Can reduce min \( q^2 \) by 30% by paying penalty of 20% increase in \( u.p \)
- mNRQCD won’t be a cure, but at least a modest tool
Lattice artifacts as boosted momenta approach inverse lattice spacing
Boosting and discretization errors

Discretization errors only double at $\nu=0.6$: least of our worries
Heavy quark with mNRQCD

Field transformation

\[ \Psi(x) = \frac{1}{\sqrt{\gamma}} S(\Lambda_v) \ T_{FW}(v) \ T_{TD}(v) \begin{pmatrix} \psi_v(x) \\ \chi_v(x) \end{pmatrix} \]

with

\[ \gamma = \frac{1}{\sqrt{1-v^2}} \]

Lorentz boost

[S(\Lambda_v)]

Foldy-Wouthuysen-Tani transformation

\[ T_{FW}(v) = \exp \left( -im \ u \cdot x \ \gamma^0 \right) \left[ 1 + \frac{i}{2m} \Lambda^\mu \gamma^j D_\mu + ... \right] \]

Remove time derivatives from Hamiltonian

\[ T_{TD}(v) = 1 + \frac{i}{4\gamma m} \gamma^0 \left[ \left( \frac{1}{1-v^2} - 1 \right) D_0 + \left( \frac{1}{1-v^2} + 1 \right) v \cdot D \right] + ... \]
mNRQCD Lagrangian

\[ \mathcal{L}_{\psi_v} = \psi_v \dagger \left[ iD_0 + H \right] \psi_v \]

\[ H_0 = i\mathbf{v} \cdot \mathbf{D} + \frac{\mathbf{D}^2 - (\mathbf{v} \cdot \mathbf{D})^2}{2\gamma m} \]

\[ \delta H = \frac{i\{\mathbf{v} \cdot \mathbf{D}, \mathbf{D}^2\} - 2i(\mathbf{v} \cdot \mathbf{D})^3}{4\gamma^2 m^2} + \frac{g(\mathbf{D}^{\text{ad}} \cdot \mathbf{E} - \mathbf{v} \cdot (\mathbf{D}^{\text{ad}} \times \mathbf{B}))}{8m^2} + \frac{(2 - \mathbf{v}^2)g(\mathbf{D}_0^{\text{ad}} - \mathbf{v} \cdot \mathbf{D}^{\text{ad}})(\mathbf{v} \cdot \mathbf{E})}{16m^2} + \frac{ig\{\mathbf{v} \cdot \mathbf{D}, \mathbf{v} \times \mathbf{E}'\}}{8\gamma m^2} - \frac{ig\{\mathbf{v} \cdot \mathbf{D}, \mathbf{v} \cdot \mathbf{E}'\}}{8(\gamma + 1)m^2} + \mathcal{O}(m^{-3}) \]

\[ \mathbf{B}' = \gamma \left( \mathbf{B} - \mathbf{v} \times \mathbf{E} - \frac{\gamma}{1 + \gamma} \mathbf{v} (\mathbf{v} \cdot \mathbf{B}) \right), \]

\[ \mathbf{E}' = \gamma \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} - \frac{\gamma}{1 + \gamma} \mathbf{v} (\mathbf{v} \cdot \mathbf{E}) \right). \]
Dispersion relation

Plots from Stefan Meinel: data for $\eta_b$ on coarse MILC lattice

$au_0m_l = 0.007$, $au_0m_s = 0.05$
Dispersion relation

Plots from Stefan Meinel: data for $\eta_b$ on coarse MILC lattice

$au_0 m_l = 0.007, au_0 m_s = 0.05$

$v_1 = 0.3$
Dispersion relation

Plots from Stefan Meinel: data for $\eta_b$ on coarse MILC lattice

$au_0 m_l = 0.007$, $au_0 m_s = 0.05$
Improved staggered quarks

- Intend to use MILC lattices, then HISQ lattices (MILC/HPQCD)
- Issues of principle: continuum limit before chiral limit
- In practice: work with light quark masses
- Fit NRQCD + staggered data with SχPT
- Vector mesons in final state: a trustworthy low energy theory? Or hope for mild mass dependence
K* mass on MILC lattices

- Unquenched data
- Communicated by D. Toussaint, MILC
- Interpolated to \((m_1, m_s)\) using \((m_1, m_1)\) & \((m_1, m_2)\)
- Discretization errors small (for our purposes)
- Negligible taste splitting between local and 1-link tastes (not shown)
ρ mass on MILC lattices

- Unquenched data

- Communicated by D. Toussaint, MILC

- Effected by $\pi - \pi$ threshold

- Discretization errors small (for our purposes)

- Negligible taste splitting between local and 1-link tastes (not shown)
Threshold effects

- An interesting problem in general
- Matrix elements are defined treating the $K^*$ as an external state
- Do the same on the lattice: keep the $K^*$ stable
- Perhaps smaller effects for $K^*$ than for $\rho$ ($K^*$ width is 50 MeV, $\rho$ width is 150 MeV)
Main worry

\[ \sigma_t \propto \sqrt{\langle C_t^2 \rangle - \langle C_t \rangle^2} \]

\[ \propto 2E_{Qq}(v)t - \left[ E_{QQ}(v = 0) + E_{qq} \right] t \]

Glasgow group report bad signal-to-noise with \( v \geq 0.2 \) when the light meson has nonzero \( p \)

But we’ve faced similar worries in other problems, sometimes successfully

Random wall sources (MILC, Kit Wong’s poster) combined with heavy quark smeared sources (K. Foley thesis)
Experience with matching NRQCD currents to $V_\mu$ and $A_\mu$

L. Khomskii finalizing 1-loop mNRQCD self-energy and completing mNRQCD matching for $V_\mu$ and $A_\mu$ through $1/m$

Work on tensor current matching just underway
Current matching

\[ \langle \text{light, } p' \mid J_4 \mid \text{heavy, } P \rangle_{QCD} = \sum_{k=1}^{10} b_k R_k \]

where

\[
\begin{align*}
R_1 &= \bar{q}(p') \gamma_5 \gamma_4 \Lambda Q(p), \\
R_2 &= \bar{q}(p') \gamma_5 \Lambda Q(p), \\
R_3 &= \frac{v \cdot p}{2m} \bar{q}(p') \gamma_5 \Lambda Q(p), \\
R_4 &= \frac{v \cdot p}{2m} \bar{q}(p') \gamma_5 \gamma_4 \Lambda Q(p), \\
R_5 &= \frac{i}{2m} \bar{q}(p') \gamma \cdot p \gamma_5 \gamma_4 \Lambda Q(p), \\
R_6 &= \frac{i}{2m} \bar{q}(p') \gamma \cdot p \gamma_5 \Lambda Q(p), \\
R_7 &= \frac{i}{m} \bar{q}(p') \gamma \cdot p' \gamma_5 \Lambda Q(p), \\
R_8 &= \frac{v \cdot p'}{m} \bar{q}(p') \gamma_5 \gamma_4 \Lambda Q(p), \\
R_9 &= \frac{i}{m} \bar{q}(p') \gamma \cdot p' \gamma_5 \gamma_4 \Lambda Q(p), \\
R_{10} &= \frac{v \cdot p'}{m} \bar{q}(p') \gamma_5 \Lambda Q(p),
\end{align*}\]
Remark on Wick rotation

- Analytic continuation prescription: fermion pole and left-most gluon pole should be inside integration contour, right-most gluon pole outside
- Known since U. Aglietti, NP B421 (1994)
- For all $\nu$, a gap between fermion and right-most gluon poles
Closing remarks

✦ Worth doing our best on form factors for rare $B$ decays
   In fact, too important to neglect

✦ Challenges = opportunities to solve interesting problems

✦ Need several approaches to check consistency
   ✦ Sum rules

✦ mNRQCD + improved staggered looks promising to us
Beyond here, there be dragons
Form factors, pseudoscalar final state

\[ \langle P(p')|\bar{q}\gamma^{\mu}b|\bar{B}(p)\rangle = f_+(q^2) \left[ p^{\mu} + p'^{\mu} - \frac{M^2 - m_P^2}{q^2}q^{\mu} \right] \]

\[ + f_0(q^2)\frac{M^2 - m_P^2}{q^2}q^{\mu}, \quad (1) \]

\[ \langle P(p')|\bar{q}\sigma^{\mu\nu}q_v b|\bar{B}(p)\rangle = \frac{if_T(q^2)}{M + m_P} \left[ q^2(p^{\mu} + p'^{\mu}) - (M^2 - m_P^2)q^{\mu} \right], \quad (2) \]
Form factors, vector final state

\[\langle V(p', \varepsilon^*) | \bar{q} \gamma_\mu b | \bar{B}(p) \rangle = \frac{2i V(q^2)}{M + m_V} \epsilon^{\mu \nu \rho \sigma} \varepsilon^*_\nu p'_{\rho} p_{\sigma}, \quad (3)\]

\[\langle V(p', \varepsilon^*) | \bar{q} \gamma_\mu \gamma_5 b | \bar{B}(p) \rangle = 2m_V A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q^\mu
\]
\[\quad + (M + m_V) A_1(q^2) \left[ \varepsilon^{* \mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right]
\]
\[\quad - A_2(q^2) \frac{\varepsilon^* \cdot q}{M + m_V} \left[ p^\mu + p'^{\mu} - \frac{M^2 - m_V^2}{q^2} q^\mu \right], \quad (4)\]

\[\langle V(p', \varepsilon^*) | \bar{q} \sigma^{\mu \nu} q_{\nu} b | \bar{B}(p) \rangle = 2T_1(q^2) \epsilon^{\mu \nu \rho \sigma} \varepsilon^*_\nu p_{\rho} p'_{\sigma}, \quad (5)\]

\[\langle V(p', \varepsilon^*) | \bar{q} \sigma^{\mu \nu} \gamma_5 q_{\nu} b | \bar{B}(p) \rangle = (-i)T_2(q^2) \left[ (M^2 - m_V^2) \varepsilon^{* \mu} - (\varepsilon^* \cdot q)(p^\mu + p'^{\mu}) \right]
\]
\[\quad + (-i)T_3(q^2)(\varepsilon^* \cdot q) \left[ q^\mu - \frac{q^2}{M^2 - m_V^2} (p^\mu + p'^{\mu}) \right], \quad (6)\]