Heavy-light matrix elements in static limit with $N_f = 2 + 1$ domain wall fermions

progress report for $f_B$ and $B_B$ on a large volume lattice

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Lattice 2007
Introduction
Babar, Belle, CDF achieved the precision measurement of
\[ \left| \frac{V_{td}}{V_{ts}} \right| = 0.2060 \pm 0.0007 \text{(exp.)}^{+0.0081}_{-0.0060} \text{(theo.)} \]

DPF2006 by Kroll (CDF)

through oscillation frequencies \( \Delta m_q \) of neutral \( B_q \) (\( q = d, s \)) mesons. Theoretical uncertainty is from lattice: (2+1f and 2f hybrid: Okamoto, lattice 2005)

- \( B_0^q \rightarrow \overline{B}_0^q \) oscillation frequency

\[
\Delta m_q = \frac{G_F^2 m_W^2}{16 \pi^2 m_{B_q}} \left| V_{tq}^* \ V_{tb} \right|^2 S_0 \left( \frac{m_t^2}{m_W^2} \right) \eta_B \mathcal{M}_q,
\]

- \( B_0^q \rightarrow \overline{B}_0^q \) mixing matrix elements

\[
\mathcal{M}_q = \left\langle \overline{B}_q^0 \left| [\bar{b} \gamma^\mu (1 - \gamma_5) q] [\bar{b} \gamma_\mu (1 - \gamma_5) q] \right| B_q^0 \right\rangle.
\]

- \( SU(3) \) breaking ratio: \( \xi = \sqrt{\frac{\mathcal{M}_s}{\mathcal{M}_d}} = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}} \)

Full 2 + 1f calculation of \( \xi \) must be carried out!
Heavy-Light matrix elements with static approximation

DWF with static heavy quark

- provides information for $m_B \to \infty$.
- Useful for precision calculation of matrix elements as a reference point for: $1/m_B$ expansion, relativistic heavy quarks....

Heavy-light projects in RBC/UKQCD collaborations

- $16^3 \times 32 \, f_B, B_B$
  - talk by Jan Wennekers (Friday)
- Lattice perturbation, theoretical issues
  - talk by Thomas Dumitrescu (Friday)
- In this talk, new developments on $24^3 \times 64$. 
Large volume for $B_B$, $f_B$ in static approximation

good and challenging

$(1.8 \text{ fm})^3$ results with APE smearing for the heavy quark action.

Good:

- new $(2.7 \text{ fm})^3$ results will provide check on finite volume effect.
- We have an ensemble with smaller $u$, $d$ quark mass, which will be useful to control chiral extrapolation, especially for $B_d$ meson.

Challenging:

- Cannot use wall source: too far plateau. Signal would be lost.
- Compact source: early plateau. But, can sample only a fraction of whole volume with 1 inversion of light quark propagator.
  (Compare: volume helps for light-light case.)
Measurement scheme

We will use/test:

- For the static quark action:
  - HYP smearing: (Alpha collaboration: smaller power divergence leads to better N/S).
    \[
    \frac{\Delta G(t)}{G(t)} \sim \exp\left[\left(m_{hl} - \frac{m_{hh} + m_{ll}}{2}\right)t\right]
    \]

- To enhance the ground state signal:
  - compact source: Gauge covariant Gaussian smearing for light or both light and heavy quark sources.
  - links in the Gaussian smearing is smeared with spatial APE smearing.
  - convolved smeared source with gauge noise will sample whole \( V \) but, how N/S goes?
Several source combinations to test

(heavy source - light source)
  • (wall - wall)
  • (point - Gauss)
  • (wall - Gauss wall) → convoluted (point - Gauss) by gauge noise
  • (Gauss - Gauss) → enhanced statistics yielded by multiple static quarks
Test of source combination on small lattice

<table>
<thead>
<tr>
<th>heavy</th>
<th>wall</th>
<th>point</th>
<th>wall</th>
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Gauss smearing uses $\sigma_s = 2$ with Gaussian width $\sigma = \sigma_s / \sqrt{2}$

- quench Wilson $\beta = 5.7 \ (a = 0.2 \text{ fm}), \ 8^3 \times 16 \ ((1.6 \text{ fm})^3)$
- domain wall fermion with $L_s = 4$, at about strange mass.

source-sink symmetric

$\langle B|VV+AA|B\rangle = \langle JOJ\rangle / \langle JJ \rangle$
Test of source combination on small lattice

| heavy | wall point wall Gauss |
| light | wall Gauss Gauss wall Gauss |

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Finding optimal Gaussian width

- use smaller volume $16^3 \times 32$, with exactly same parameters as $24^3 \times 64$, except $L_s = 16 \rightarrow 4$, $m_f = 0.02$.

Gaussian-Gaussian -- symmetric sink

- $\sigma_s = 4$
Finding optimal number of iteration

For $\sigma_s = 4$

Computational cost for the Gaussian smearing is not negligible. Find smallest number of iteration giving same result as larger number iteration.

Gaussian - Gaussian, $\sigma_s = 4$

\[
< B|V+AA|B> \propto <JOJ>/\sqrt{<JJ><JJ>}
\]

$N = 32$

\[
\left\langle J_B(t_{\text{sink}}) O(t) J_B(t_{\text{src}}) \right\rangle
\sqrt{\left\langle J(t_{\text{sink}}) J(t) \right\rangle \left\langle J(t) J(t_{\text{src}}) \right\rangle}
\]
Some results (preliminary): $\Phi_q = f_{Bq} \sqrt{m_{Bq}} [\text{GeV}^{3/2}]$

On $24^3 \times 64$, $L_s = 16$ lattices

Using 1-loop lattice perturbation theory and NLO matching to QCD:
Some results (preliminary): $a^3 M_q/m_B$

On $24^3 \times 64$ lattices

\[ \frac{\langle J_B(t_{sink}) O(t) J_B(t_{src}) \rangle e^{m^*(t_{sink} - t_{src})/2}}{\sqrt{\langle J_B(t_{sink}) J_B(t) \rangle \langle J_B(t) J_B(t_{src}) \rangle}} \]
Systematic error from chiral-disallowed mixing among various systematic errors

- finite $m_{\text{res}}$ (from finite $L_s$) allows transition of $R \leftrightarrow L$ of light quark with rate $m_{\text{res}}a$.

$$
O_{VV+AA} \propto \bar{b}_L \sigma_{\mu} d_L \bar{b}_L \sigma_{\mu} d_L + \bar{b}_R \bar{\sigma}_{\mu} d_R \bar{b}_R \bar{\sigma}_{\mu} d_R \\
O_{VV- AA} \propto \bar{b}_L \sigma_{\mu} d_L \bar{b}_R \bar{\sigma}_{\mu} d_R \\
O_{SS- PP} \propto \bar{b}_L d_R \bar{b}_R d_L \\
O_{SS+PP} \propto \bar{b}_L d_R \bar{b}_L d_R + \bar{b}_R d_L \bar{b}_R d_L
$$

- mixing of $SS + PP$ to $VV + AA$ is disallowed by heavy quark symmetry (Becirevic et al, and ALPHA collaboration).

- mixing of $VV - AA$ or $SS - PP$ to $VV + AA$ is suppressed at least by a factor $m_{\text{res}}a \approx 0.3\%$. Thus, negligible in the present accuracy.
Summary

- To compute $\mathcal{M}_q = \langle \bar{B}_q^0 | \bar{b} \gamma^\mu (1 - \gamma^5) q | \bar{b} \gamma^\mu (1 - \gamma^5) q | B_q^0 \rangle$ and $f_B$ on a large volume lattice, several source methods have been tested.

- No cheap trick tested was useful. The best source is smearing both light and heavy quarks. (Though it does not exclude yet another cheap trick that might work.)

- Optimal parameter for the Gaussian smearing was found.

- Preliminary results are consistent with the small volume results with much larger statistics.

- Need to keep accumulate the statistics. Smaller mass point will be examined, which will help reduce the error in the chiral extrapolation.

- Chirally disallowed operator mixing was discussed. It was found to be negligible.