Nucleon form factors with $N_f=2+1$ domain wall fermions

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Outline

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   - vector form factors
   - axial vector form factors

4. Summary

Related talk: S. Ohta, nucleon structure functions, 7/31(Tue) 17:50
1. Introduction

Motivation: understand nucleon physics from first principle

We calculate matrix elements related to nucleon iso-vector vector and axial vector form factors on $N_f = 2 + 1$ domain wall fermion (DWF) configuration.

Form factors (Nucleon elastic scattering)

- Vector form factors

\[
\langle N, p|V_\mu(x)|N, p' \rangle = \bar{u}_N(p) \left( F_1(q^2) \gamma_\mu + i\sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2M_N} \right) u_N(p') e^{iqx}
\]

$F_1(q^2), F_2(q^2) \rightarrow \langle r^2_1 \rangle, \langle r^2_2 \rangle$ related to charge radii $\langle r^2_p \rangle, \langle r^2_n \rangle$

$F_2(0) = \mu_p - \mu_n - 1$

- Axial vector form factors

\[
\langle N, p|A_\mu(x)|N, p' \rangle = \bar{u}_N(p) \left( G_A(q^2) i\gamma_5\gamma_\mu + i\gamma_5q_\mu G_P(q^2) \right) u_N(p') e^{iqx}
\]

$G_A(q^2), G_P(q^2) \rightarrow g_A/g_V, \langle r^2_A \rangle, g_{\pi NN}, g_P$
2. Simulation parameters

- $N_f = 2 + 1$ Iwasaki gauge + Domain Wall fermion actions
- $\beta = 2.13 \ a^{-1} = 1.73 \ \text{GeV} \ M_5 = 1.8 \ m_{\text{res}} \approx 0.003$
- Lattice size $24^3 \times 64 \times 16 \ (La \approx 2.7 \ \text{fm})$
- $m_s = 0.04$ fixed (close to $m_s^{\text{phys}}$)
- quark masses $m_f = m_{\text{sea}} = m_{\text{val}}$ and confs.

<table>
<thead>
<tr>
<th>$m_f$</th>
<th>$m_\pi$[MeV]</th>
<th># of confs.</th>
<th>$N_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>330</td>
<td>360</td>
<td>4</td>
</tr>
<tr>
<td>0.01</td>
<td>420</td>
<td>180</td>
<td>4</td>
</tr>
<tr>
<td>0.02</td>
<td>560</td>
<td>98</td>
<td>4</td>
</tr>
<tr>
<td>0.03</td>
<td>670</td>
<td>106</td>
<td>4</td>
</tr>
</tbody>
</table>

- Gaussian smearing is employed.
- Matrix elements are evaluated by ratio of 3- and 2-point functions.
- We focus only on iso-vector quantities. (no disconnected diagram)
- Four different non-zero $q^2$ with $(pL/2\pi)^2 = 1, 2, 3, 4$
- All results are preliminary.
3. Results
3.1. $g_A/g_V = G_A(0)/F_1(0)$

Lightest result is smaller than other mass points. This might be caused by large finite volume effect, because similar tendency was seen in quenched calculation in 1.2fm volume.
3.1.1. $g_A/g_V$ with DWF

\[ N_f = 0 \text{ RBCK, } N_f = 2 \text{ RBC, } N_f = 2 + 1 \text{ RBC-UKQCD} \]

$N_f = 2(1.9\text{fm})$ has similar $m_{\pi}^2$ dependence as $N_f = 2 + 1(2.7\text{fm})$.

Strange behavior happens at heavier region in smaller volume.
3.1.2. $g_A/g_V$ vs $m_\pi L$ with DWF

$N_f = 0$ RBCK, $N_f = 2$ RBC, $N_f = 2 + 1$ RBC-UKQCD

$N_f = 2 + 1 (2.7 \text{fm})$ drops at $m_\pi L \sim 4.5$ as well as $N_f = 2 (1.9 \text{fm})$.

$N_f = 0 (2.4 \text{fm})$ does not decrease as $m_\pi$ decreases.

It might be insensitive finite volume effect due to lack of sea quarks.
3.1.2. $g_A/g_V$ vs $m_\pi L$ with DWF

$N_f = 0$ RBCK, $N_f = 2$ RBC, $N_f = 2 + 1$ RBC-UKQCD
$N_f = 2 + 1(1.8\text{fm})$ might have similar trend as $N_f = 2(1.9\text{fm})$ and $N_f = 2 + 1(2.7\text{fm})$ except lightest $m_\pi$ with larger error.

Further investigation of finite volume effect is necessary.

$m_\pi L \sim 4.5$ seems threshold.
3.1.3. $g_A/g_V$ vs $m_\pi L$ with dynamical calculation

Results with Wilson fermion also have similar trend.

Estimation of threshold $m_\pi L \sim 4.5$ is not so bad.
3.1.4. Chiral extrapolation of $g_A/g_V$

![Graph showing $g_A/g_V$ versus $m_{\pi}^2$ (GeV^2)]

**Linear chiral extrapolation without lightest result**

$$g_A/g_V = \begin{cases} 
1.220(85) \text{ (lat.)} \\
1.270(3) \text{ (exp.)}
\end{cases}$$
3.2 Iso-vector vector form factors

\[ F_1(q^2) \]

\[ F_2(q^2) \]

Renormalized by \( Z_V = 1/F_1(0) \)

\( F_1(q^2) \) is almost independent of quark mass except lightest result.

\( F_2(q^2) \) has some quark mass dependence, but it is not monotonic.
3.2.1 Dipole fit of vector form factors

1 parameter fit $M_1$

$$F_1(q^2) = \frac{1}{(1 + q^2/M_1^2)^2}$$

$\sqrt{\langle r_1^2 \rangle} = \sqrt{12}/M_1$

2 parameters fit $F_2(0)$ and $M_2$

$$F_2(0) = \frac{\mu_p - \mu_n - 1}{(1 + q^2/M_2^2)^2}$$

$\sqrt{\langle r_2^2 \rangle} = \sqrt{12}/M_2$
3.2.2 Iso-vector Dirac rms radius $\sqrt{\langle r_1^2 \rangle}$

Result approaches to experiment as $m_\pi$ decreases. However, lightest result might be affected by large finite volume effect. To check reliability of lightest result, we need more detailed finite volume study of this form factor.
3.2.3 $F_2(0)$ and Iso-vector Pauli rms radius $\sqrt{\langle r_2^2 \rangle}$

![Graph showing $F_2(0)$ and $(\langle r_2^2 \rangle)^{1/2}$ vs $m_\pi^2$.[Image]]

$F_2(0) = \mu_p - \mu_n - 1$

Finite volume might effect lightest results.

$F_2(0)$ is comparable to experiment.

$\sqrt{\langle r_2^2 \rangle}$ approaches to experiment from heavier $m_\pi$ than $\sqrt{\langle r_1^2 \rangle}$.

We will improve statistics at $m_f = 0.01$ to confirm $m_\pi$ dependence.
3.3. Iso-vector axial vector form factors

Both are renormalized by $Z_V = 1/F_1(0) \approx Z_A$.

$G_A(q^2)$ is almost independent of quark mass except lightest result. Lightest result of $G_A(q^2)$ is smaller than other results and would include large finite volume effect.

$G_P(q^2)$ at smallest $q^2$ increases as quark mass decreases except lightest result. This trend is consistent with pion pole dominance of $G_P$. 
3.3.1. Dipole fit and Goldberger-Treiman relation

1 parameter fit \( M_A \)

\[
\frac{G_A(q^2)}{G_A(0)} = \frac{1}{(1 + q^2/M_A^2)^2}
\]

generalized G-T relation

\[
G_P(q^2) = \frac{2m_N G_A(q^2)}{q^2 + m_\pi^2}
\]

We evaluate axial charge rms radius \( \sqrt{\langle r_A^2 \rangle} = \sqrt{12}/M_A \).

Ratio based on G-T relation is almost insensitive to \( q^2 \). \( G_P \) would be explained by \( G_P(q^2) = \alpha_P \times 2m_N G_A(q^2)/(q^2 + m_\pi^2) \).
3.3.2 Iso-vector axial charge rms radius $\sqrt{\langle r_A^2 \rangle}$

Lightest result might have large finite volume effect, but there is tendency to approach experiment. We need to improve statistics at $m_f = 0.01$. 

3.3.3 \( g_{\pi NN} \) coupling and \( g_P \) for muon capture

G-T relation \( g_{\pi NN} = m_N g_A / f_\pi \)

\[
m_N g_A / f_\pi = m_\mu G_P(0.88m_\mu^2)
\]

\( f_\pi \) is fixed by experiment.

\[
\text{GT(meas. } \alpha_P \text{)} : \alpha_P \times 2m_N G_A(0.88m_\mu^2)
\]

\[
G_P(q^2 + m_\pi^2) : \text{dipole fit of } G_P(q^2)(q^2 + m_\pi^2)
\]

Both lightest results are omitted in chiral extrapolation due to large finite volume effect.

Results obtained from extrapolation agree with experiment in both cases.
4. Summary

- We calculated nucleon matrix elements with $N_f = 2 + 1$ dynamical domain wall fermions at four quark masses.
- Lightest results would include large finite volume effect, e.g., $g_A/g_V$, $G_A(q^2)$, and $G_P(q^2)$. Threshold $m_\pi L \sim 4.5$ for $g_A/g_V$. We need more detailed finite volume study.
- While all results are preliminary, we found encouraging results.

Future work

- Improve statistics at $m_f = 0.01$
- Finite volume study with smaller volume result
Backup Slides
Finite volume study of $g_A/g_V = G_A(0)/F_V(0)$
Well determined in experiment: $g_A/g_V = 1.2695(29)$

$g_A/g_V$ is simple, basic physical quantity in nucleon matrix element.

It is easy to calculate with DWF due to $Z_V/Z_A \approx 1$.

Large finite volume effect is seen in heavy $m_\pi$ region.

$g_A/g_V$ at 2.4 fm agrees well with one at 3.6 fm.

$L \approx 2.5$ fm seems to be enough in this quenched calculation.
Each component of $g_A/g_V$

$g_V^{\text{lat}}$ has reasonable $m_{\pi}^2$ dependence, and seems to agree with $Z_A^{-1}$ in chiral limit within 1%.

$g_A^{\text{lat}}$ decreases at lightest pion mass.

Axial vector form factor has strange pion mass dependence.
\[ g_A / g_V \]

N\(_f\) = 2+1 2.7 fm
N\(_f\) = 2 1.9 fm
N\(_f\) = 0 3.6 fm

experiment
N\(_f\) = 2+1 2.5 fm LHPC
N\(_f\) = 2+1 3.5 fm LHPC

\[ m_\pi^2 [\text{GeV}^2] \]
\begin{align*}
\text{m} & \quad \pi \\
\text{L} \\
\text{0.8} & \quad \text{0.9} & \quad \text{1} & \quad \text{1.1} & \quad \text{1.2} & \quad \text{1.3} & \quad \text{1.4} & \quad \text{1.5} \\
N_f = 0 & \ (2.4 \text{fm}) \\
N_f = 2 & \ (1.9 \text{fm}) \\
N_f = 2+1 & \ (2.7 \text{fm}) \\
N_f = 2 & \ (1.4 \text{fm}) \text{ Wilson}^1 \\
N_f = 2 & \ (1.7 \text{fm}) \text{ Wilson}^2
\end{align*}

\begin{align*}
g_A / g_V
\end{align*}
Experiment

\( \frac{m_N g_A}{f_\pi} (\text{exp. } f_\pi) \)

\( \frac{m_N g_A}{f_\pi} (\text{meas. } f_\pi) \)

chiral limit

\( g_{\pi NN} \)
\[
\left( \langle r_A^2 \rangle \right)^{1/2} \text{[fm]}
\]

- $G_A$
- $G_P(m_\pi^2 + q^2)$

Experiment

\[m_\pi^2 \text{[GeV}^2]\]
Matrix elements

\[ R_{\vec{p}}^{PO}(t, t_{snk}, t_{src}) = \frac{G_{\vec{p}}^{PO}(t)}{G_0^{G}(t_{snk})} \left[ \frac{G_L^{G}(t_{snk} - t + t_{src})G_0^{G}(t)G_L^{L}(t_{snk})}{G_L^{G}(t_{snk} - t + t_{src})G_0^{L}(t)G_L^{L}(t_{snk})} \right]^{1/2} \]
\[ \propto \langle N(0)|O(q)|N(p)\rangle \quad (t_{src} \ll t \ll t_{snk}) \]

Normalization of operator is canceled out.

\[ G_{\vec{p}}^{PO} : \quad 3\text{-point function of } O \text{ with } \vec{p} \text{ and projector } P \]
\[ \text{gauss smearing source and local sink are employed.} \]

\[ G_{\vec{p}}^{G,L} : \quad 2\text{-point function with gauss smearing}(G) \text{ and local}(L) \text{ sink} \]