Thermodynamic properties of QCD in external magnetic fields: an update

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Motivation: Strong Magnetic fields

- early universe: $\sqrt{eB} \approx 2 \text{ GeV}$
- RHIC/LHC: 0.1..0.5 GeV, QCD scale!
  - non-central collisions
  - charged spectators
  - $B$ perp. to reaction plane
- neutron stars, magnetars: 1 MeV, $B \approx 10^{14} \text{ G}$
- cf. strongest field in lab: $10^5 \text{ G}$
  - (10$^7 \text{ G}$ unstable)
- refrigerator magnet: 100 G
- earths magn. field: 0.6 G
idealized: constant external magn. field $B$ + QCD in equilibrium

lattice: abelian space-dependent phases on the links mimicking $A_\mu$

$B$ quantized and bounded, but no sign problem

simulations similar to transition studies at $B = 0$

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)e$
- physical pion masses
- $B$ does not alter scale setting $a(\beta)$ and line of constant physics $m(\beta)$

state-of-the-art: $\sqrt{eB} = 0.1 \ldots 1$ GeV
Main result: QCD phase diagram with magnetic field

(pseudo-) $T_c$ as a function of magnetic field:

\[ \Rightarrow T_c \text{ decreases by } O(10) \text{ MeV for } eB \lesssim 0.5 \text{ GeV}^2 \]
\[ \Rightarrow \text{transition not becoming stronger} \]

strange number susceptibility
light quark condensate
both renormalized

\[ eB \]
(light) condensate and Polyakov loop as a function of $T$ at fixed $B$'s:

$T = 0$: Magnetic Catalysis

- Model Conjecture
- Monte Carlo ✓ D’Elia et al. 10

Inverse Magnetic Catalysis $\Leftrightarrow T_c \downarrow$

- Important: Masses of Current quarks
- Investigate Monte Carlo configurations!

$\Rightarrow$ sea and valence effects and P-loop

& MC and IMC in the gluon sector

& anisotropies
Magnetic catalysis

- change of condensate \[
\left[ \langle \bar{\psi} \psi_{u,d}(B) \rangle - \langle \bar{\psi} \psi_{u,d}(0) \rangle \right] m_{u=d} \text{ at } T = 0:
\]

![Graph showing change of condensate with magnetic field strength eB.]

- with \( \chi \text{PT, NJL} \)
- well approximated unless \( eB > 0.1, 0.3 \text{ GeV}^2 \)

\[ \Delta \ldots = \ldots(B) - \ldots(0) \text{ removes additive divergences (as } T) \]

\[ m \cdot \ldots \text{ removes multiplicative divergences} \]
Magnetic catalysis

- change of condensate and gluonic action:

  \[ m_{ud} \Delta \bar{\psi}_d \psi_d \text{ (GeV)} \times 10^3 \]

  \[ -\Delta I_g \text{ (GeV)} \times 10^3 \]

  \[ eB \text{ (GeV}^2) \]

  \[ T=0 \]

  very similar shape

  \[ \Rightarrow \text{gluons inherit magnetic catalysis from quarks via strong coupling} \]

  magnitude \( \mathcal{O}(100) \) larger for gluons, but \( B = 0 \) scale (= gluon condensate) already \( \mathcal{O}(200) \) larger: relative effect larger on quarks
Intermezzo: Trace anomaly

\[ I = \epsilon - 3p \quad \ldots \text{interaction measure, since free gas: } \epsilon = 3p \]

\[ \text{lattice} = - \frac{T}{V} \frac{d \log Z}{d \log a} \quad \ldots \text{scale anomaly} \]

\[ = - \frac{T}{V} \left( \frac{\partial \log Z}{\partial a} \frac{\partial \beta}{\partial \log Z} + \frac{\partial \log Z}{\partial \log am} \frac{\partial \log am}{\partial \log a} \right) \quad \beta = \frac{6}{g^2} \]

\[ = - \left( \langle s_g \rangle \frac{-\partial \beta}{\partial \log a} + m \langle \bar{\psi}\psi \rangle \frac{\partial \log am}{\partial \log a} \right) \]

\[ \Delta I = - \left( \Delta \langle s_g \rangle \frac{-\partial \beta}{\partial \log a} + m \Delta \langle \bar{\psi}\psi \rangle \frac{\partial \log am}{\partial \log a} \right) \]

⇒ change of gluonic action density and condensate go together with beta and gamma function [LCP]

!? similarity in \( B \)-dependence

(used to improve convergence of gluonic contribution)
Inverse magnetic catalysis

Again change of condensate and gluonic action, now finite $T$:

- Non-monotonic behaviour, again similar shape for quarks and gluons

$\Rightarrow$ magnetic catalysis turns into inverse magnetic catalysis around $T_c$

- Physical quark masses essential
  - Higher masses in other simulations
  - Missed in most non-lattice approaches

D’Elia et al. 10, Ilgenfritz et al. 12
Inverse magnetic catalysis: mechanism

\[ \langle \bar{\psi} \psi \rangle_{\text{full}} = \frac{\int e^{-S_g} \det(\mathcal{D}[B] + m) \text{tr}(\mathcal{D}[B] + m)^{-1}}{\int e^{-S_g} \det(\mathcal{D}[B] + m)} \]

\[ \langle \bar{\psi} \psi \rangle_{\text{val}} = \frac{\int e^{-S_g} \det(\mathcal{D}[0] + m) \text{tr}(\mathcal{D}[B] + m)^{-1}}{\int e^{-S_g} \det(\mathcal{D}[0] + m)} \]

\[ \langle \bar{\psi} \psi \rangle_{\text{sea}} = \frac{\int e^{-S_g} \det(\mathcal{D}[B] + m) \text{tr}(\mathcal{D}[0] + m)^{-1}}{\int e^{-S_g} \det(\mathcal{D}[B] + m)} \]

to lowest order approx. : \( \langle \bar{\psi} \psi \rangle_{\text{full}} \simeq \langle \bar{\psi} \psi \rangle_{\text{val}} + \langle \bar{\psi} \psi \rangle_{\text{sea}} \)

D’Elia, Negro 11
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\[ \langle \bar{\psi} \psi \rangle_{\text{sea}} = \frac{\int e^{-S_g} \det(\mathcal{D}[B] + m) \text{tr}(\mathcal{D}[0] + m)^{-1}}{\int e^{-S_g} \det(\mathcal{D}[B] + m)} \]

\[ \Delta \langle \bar{\psi} \psi \rangle \text{ at low } T \text{ and around } T_c: \]

\[ \Delta \langle \bar{\psi} \psi \rangle \simeq \langle \bar{\psi} \psi \rangle_{\text{val}} + \langle \bar{\psi} \psi \rangle_{\text{sea}} \]

D’Elia, Negro 11

FB, Endrődi, Kovács 13
$\mathcal{D}$ has more small eigenvalues with $B$

- in valence trace $\downarrow$
- generates condensate
- = statement about the change of the spectrum (even quenched)

$\downarrow$ in sea determinant
- leads to a $B$-dep. probability
- = statement about the typical gauge field
- = feedback of quarks

Sea effect is particularly effective near $T_c$! Why increasing at low $T$?

Washed out for heavy quarks

Increasing Polyakov loop indicates change of typical gauge field
Anisotropy I: Magnetic susceptibility

- tensor polarization:
  \[
  \langle \bar{\psi} \sigma_{\mu\nu} \psi \rangle \propto F_{\mu\nu} \quad \langle \bar{\psi} \sigma_{12} \psi \rangle \equiv qB \underbrace{\langle \bar{\psi} \psi \rangle \cdot \chi}_{\tau} + O((qB)^3)
  \]

for each quark flavor

- \(\chi\) relevant for radiative \(D_s\) meson transitions, anomalous magn. moment of the muon, chiral-odd photon distribution amplitudes
- \(\tau\) like order parameter, physical values
- \(\tau\) in free energy:
  \[
  F \propto -\tau(qB)^2 + \text{angular momentum} + O((qB)^4)
  \]

negative \(\tau\) as we find \(\Rightarrow\) free energy increases with \(B \Rightarrow\) diamagnetic contribution from spin

but: total \(O(B^2)\) at \(T = 0\) completely fixed by charge renormalization
Anisotropy II: Field strength components

plaquette expectation values in various planes:

\[ \langle \text{tr} \mathcal{E}^2 \rangle < \langle \text{tr} \mathcal{E}^2 \rangle^T = 0 < \langle \text{tr} \mathcal{B}^2 \rangle < \langle \text{tr} \mathcal{B}^2 \rangle^\perp < \langle \text{tr} \mathcal{B}^2 \rangle^\parallel \]

\[ A(\mathcal{E}) \equiv \langle \text{tr} \mathcal{E}^2 \rangle - \langle \text{tr} \mathcal{E}^2 \rangle^\parallel > 0 \quad A(\mathcal{B}) \equiv \langle \text{tr} \mathcal{B}^2 \rangle^\perp - \langle \text{tr} \mathcal{B}^2 \rangle^\parallel < 0 \]

same for coarse and heavy \( N_f = 2 \) simulations

in line with perturbative Euler-Heisenberg eff. Lagrangian:

\[
S_{\text{eff}} = - \log \det(\mathcal{D}[B] + m) \sim \frac{(qB)^2}{m^4} \left[ \frac{5}{2} \langle \text{tr} \mathcal{E}^2 \rangle - \langle \text{tr} \mathcal{B}^2 \rangle^\perp - \langle \text{tr} \mathcal{E}^2 \rangle^\parallel - 3 \langle \text{tr} \mathcal{B}^2 \rangle^\parallel \right]
\]
Anisotropy III: Quark sector

- quark action:

\[ A_f \equiv \langle \bar{\psi}_f D_\perp \psi_f \rangle - \langle \bar{\psi}_f D_\parallel \psi_f \rangle \text{ for all flavors} \]

\[ \Rightarrow \text{negative} \]

\[ \Rightarrow \text{bigger than gluonic anisotropy} \]

\[ \Rightarrow \text{roughly independent of temperature} \]
Anisotropy IV: Topological charge

- two-point correlator in different directions:

\[ \langle q(0)q(r) \rangle_{\vec{r} \parallel \vec{B}} \text{ vs. } \langle q(0)q(r) \rangle_{\vec{r} \perp \vec{B}} \text{ vs. } \langle q(0)q(r) \rangle \]

\[ \Rightarrow \text{ no stat. significant anisotropy} \]

although topology related to quark zero modes (index theorem) and those should become elongated along \( B \) (Landau levels) . . .
(An)isotropic pressure and magnetization

Pressure $p$ is change of free energy under compression of volume, now sensitive to direction:

$$p_i = -\frac{L_i}{V} \frac{dF}{dL_i}$$
(An)isotropic pressure and magnetization

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- Specify whether magn. field $eB$ or magn. flux $\Phi = eB \cdot L_x L_y$ constant

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- Specify whether magn. field $eB$ or magn. flux $\Phi = eB \cdot L_x L_y$ constant

Physics question, e.g. $\Phi$-scheme: perfect conductors, field lines frozen in

- Homogeneous system: free energy extensive with density $f$

$$F = L_x L_y L_z \cdot f(eB) = L_x L_y L_z \cdot f\left(\frac{\Phi}{L_x L_y}\right)$$

Additional $L_{x,y} = L_\perp$-dependence $\Rightarrow$

$$p_{\perp}^{(B)} = p_{\|}^{(B)} = p_{\|}^{(\Phi)} \neq p_{\perp}^{(\Phi)}$$

Isotropic pressure = Anisotropic pressure

Not very clear in the literature (!) on the lattice fortunately . . .
...we’ve got a quantized magn. flux and thus the $\Phi$-scheme, moreover:

$$p_{\perp}^{(\Phi)} - p_{\|}^{(\Phi)} = - \frac{\partial f}{\partial eB} \cdot L_{\perp} \frac{\partial eB}{\partial L_{\perp}} \bigg|_{\Phi=\text{const.}} eB \propto L_{\perp}$$

$$= - M \cdot eB$$

$\Rightarrow$ measure the magnetization $M$
...we’ve got a quantized magn. flux and thus the $\Phi$-scheme, moreover:

$$p_{\perp}^{(\Phi)} - p_{\parallel}^{(\Phi)} = -\frac{\partial f}{\partial eB} \cdot L_{\perp} \frac{\partial eB}{\partial L_{\perp}} \bigg|_{\Phi={\text{const.}}} \Rightarrow \phi = \text{const.: } eB \propto L_{\perp}$$

$$= -M \cdot eB$$

⇒ measure the magnetization $M$

via anisotropic lattices, schematically:

$$a_{\parallel} = \xi a_{\perp} \xrightarrow{N_{\mu} \text{ const.}} L_{\parallel} = \xi L_{\perp}$$

$$S = \xi_{g,0} \cdot \text{plaq}_{\parallel} + \xi_{g,0}^{-1} \cdot \text{plaq}_{\perp} + \xi_{f,0} \cdot \bar{\psi}_f D_{\parallel} \psi_f + \bar{\psi}_f D_{\perp} \psi_f + \bar{\psi}_f m_f \psi_f$$

$\xi$ is the physical anisotropy, $\xi_{.,0}(\beta)$ are bare anisotropies = parameters to be tuned in the continuum limit along a line of fixed $\xi$

⇒ (bare) magnetization through gluonic and quark anisotropies:

$$-M \cdot eB = -\zeta_g [A(E) - A(B)] - \zeta_f \sum_f A_f$$
with coefficients:

\[ \zeta. \sim \left. \frac{\partial \xi_{..0}}{\partial \xi} \right|_{\xi=1} \xrightarrow{\text{pert.}} 1 + \mathcal{O}(g^2) \]

First approx. \( \zeta = 1 \) and subtraction of \( O(B^2) \) term (charge renorm.):

\[ \Rightarrow QCD \, \text{vacuum is paramagnetic} \]

Including result from Hadron Resonance Gas
Summary

- phase diagram: $T_c(B)$ decreases by $O(10)$ MeV, still crossover
- Magnetic Catalysis: $\langle \bar{\psi} \psi \rangle(B) \uparrow$ at zero temperature
  - $\uparrow$ also for gluons, cf. trace anomaly
- Inverse Magnetic Catalysis: $\langle \bar{\psi} \psi \rangle(B) \downarrow$ near transition
  - quark back reaction: sea effect dominates, only at phys. masses
  - Polyakov loop $\uparrow$
- anisotropies:
  - magn. susceptibility: $\langle \bar{\psi} \sigma_{\mu\nu} \psi \rangle \propto B$
  - field strength: $\langle \text{tr} E^2 \parallel \rangle < \langle \text{tr} E^2 \perp \rangle$, $\langle \text{tr} B^2 \parallel \rangle < \langle \text{tr} B^2 \perp \rangle$
    - cf. perturbative Euler-Heisenberg
  - quark action: $\langle \bar{\psi}_f D \parallel \psi_f \rangle > \langle \bar{\psi}_f D \perp \psi_f \rangle$, dominates
  - topology: no anisotropy in correlator
- (an)isotropic pressure:
  - depends on scheme: fixed field vs. flux
  - allows to compute on the lattice magnetization from anisotropies:
    - QCD vacuum paramagnetic (here renorm. still to perform)
Backup: more simulation details

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges \((q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e\)
- lattice spacing set at \(T = 0, B = 0\)
  - physical pion masses
    - set by \(f_K, f_K/m_\pi\) and \(f_K/m_K\)
- \(T = 0\): \(24^3 \times 32, 32^3 \times 48\) and \(40^3 \times 48\) lattices
- \(T > 0\): \(N_t = 6, 8, 10\) meaning \(a = 0.2, 0.15, 0.12\) fm
  - \(N_s = 16, 24, 32\) for finite volumes
- condensates from stochastic estimator method with 40 vectors
- magn. flux quanta: \(N_B \leq 70 < \frac{N_x N_y}{4} = 144\)
Backup: Nature of the transition

• volume dependence of light susceptibility:

\[ \chi_u a^2 \]

\[ \frac{eB}{T^2} \approx 82 \]

no volume scaling \( \Rightarrow \) remains a crossover up to \( \sqrt{eB} \approx 1 \) GeV
Backup: Mass sensitivity

- what if we put \((m_{\text{light}}, m_{\text{light}}, m_{\text{strange}}) \rightarrow (m_{\text{strange}}, m_{\text{strange}}, m_{\text{strange}})\)?

\[ T \text{-dep. of } u\text{-susceptibility (top) and change of } u\text{-condensate (bottom)} \]

\[ \Rightarrow \text{effects of decreasing } T_c \text{ & inverse magn. catalysis disappear} \]

light quark masses are important
at $T = 0$ and $\overline{\text{MS}}$ scheme at 2 GeV:

\[
\begin{align*}
\tau_{\text{up}} &= -(40.7 \pm 1.3) \text{ MeV} \\
\tau_{\text{down}} &= -(39.4 \pm 1.4) \text{ MeV} \\
\tau_{\text{strange}} &= -(53.0 \pm 7.2) \text{ MeV} \\
\chi_{\text{up}} &= -(2.08 \pm 0.08) \text{ GeV}^{-2} \\
\chi_{\text{down}} &= -(2.02 \pm 0.09) \text{ GeV}^{-2}
\end{align*}
\]

quenched unrenorm.: $\tau_{\text{up}}/\text{down} = -52 \text{ MeV}$  
Braguta, Buividovich et al. 10

QCD sum rules: $\chi_{\text{light}} = -(2.11 \pm 0.23) \text{ GeV}^{-2}$
Ball, Braun, Kivel 03

vector dominance: $\chi_{\text{light}} = -\frac{2}{m_{\rho}^2} \approx -3.3 \text{ GeV}^{-2}$

at finite $T$: $|\tau_{\text{light}}|$ decreases like an order parameter

inflection point: $T_c = 162(3)(3) \text{ MeV}$ (compatible with $T_c^{\langle \bar{\psi} \psi \rangle}$ at $B = 0$)