Implications of heavy glueball results from lattice QCD for the PANDA experiment.

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Overview of the talk

Plan of the talk.

Science Introduction to glueballs.
Science Experimental searches for glueballs.
Hope The PANDA experiment.
Speculation Why search for heavy Glueballs?¹?
Ranting Some philosophical lessons from model studies of glueballs.

In some sense this talk is an errata for my talk on glueballs at the lattice 2007 conference.

¹Not my speculations I might add.
Glueballs are bound states of glue.

The holy grail of hadron spectroscopy is to find experimental evidence for glueballs. \(^2\)

Unfortunately, glueballs will probably mix with quark-antiquark states with the same quantum numbers. **Or do they?**

Also in unquenched QCD the glueball will decay with the strong force, hence resonance effects need to be considered.

\(^2\)Our new goal after finding the Higgs and no BSM
Ed Witten on glueballs

There is a very interesting paper by Witten on gauge theory and glueballs (arXiv:0812.4512).

- Glueballs don’t exist in the classical gauge theory.

People often say to me:

I don’t understand how glueballs can be massive, if the gluons are massless.

but Ed Witten says

I have spent most of my career wishing that we had a really good way to quantitatively understand the mass gap in four dimensional gauge theory.
Main difficulties with lattice glueball studies

- The correlators have a poor signal to noise ratio. High statistics are required or better techniques.
- Strong lattice spacing dependence.

(a) Effective mass

(b) Cont. extrapolation, hep-lat/9704011
The quenched glueball spectrum

In pure gauge theory there is rich spectrum of glueball states. From Morningstar and Peardon (hep-lat/9901004). This is an iconic and perhaps the famous graph from lattice QCD. Statistical errors below error from $r_0$. Implications of heavy glueball results from lattice QCD for the
Searching for $0^{++}$ glueball

Chen et al. hep-lat/0510074, $M_{0^{++}} = 1710(50)(80)$ MeV

<table>
<thead>
<tr>
<th>Meson</th>
<th>$M$ MeV</th>
<th>$\Gamma$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0(400)$ ((\sigma))</td>
<td>400 - 550</td>
<td>400 - 700</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>980</td>
<td>40 - 100</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>1200-1500</td>
<td>200-500</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>1505</td>
<td>109</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td>1724</td>
<td>137</td>
</tr>
</tbody>
</table>

**Table:** Light $0^{++}$ mesons from PDG

- Need glueball, $\bar{q}q$ and two meson interpolating operators, at zero and non-zero momentum.
- How many exponentials needed to see glueball degrees of freedom.
- High statistics required.
The PANDA experiment and glueballs

The PANDA experiment (collaboration of 400 people) is part of FAIR at GSI. It has a broad research program (see 0903.3905). The glueball part of it is:

- It is a fixed target experiment colliding proton with anti-proton, hence should be glue friendly.
- They can tune the energy of the beams (so they will appreciate accurate masses).
- Search for glueballs with mass between 2.2 and 5.5 GeV.
- There are studies of the decay of glueballs with exotic $J^{PC} = 0^{+-}$ and $2^{+-}$ with decay width of 10 MeV!
- PANDA will probably start taking data in 2018.
Heavy Glueballs

- It would be better if a pure glueball is detected.
- The PANDA work book (0903.3905) suggests that it is easier to find heavier glueballs than light glueballs. I find this surprising.
- There are speculations (hep-ph/9906293) that there are no light mesons above 3.1 GeV.
- According to Morningstar and Peardon there are 9 glueball candidates above 3.1 GeV.
- The mass of the heaviest light meson in the 2010 PDG is $f_6(2510)$ with a mass of 2.469 GeV, but perhaps there were no experiments or the widths were too large.
- The hadron spectrum collaboration go upto 2.8 GeV.
- Hagedorn conjectured that the density of light hadrons goes like $e^{m/T}$ (1107.2130),
- There are $\bar{c}c$ mesons above 3 GeV, but Page (hep-ph/0107016) suggests that because the charm quark is heavy the mixing between glueball and $\bar{c}c$ states may be smaller than for light fermions.
Screened static potential

From hep-ph/9906293. If the string snaps like this then the spectrum will be bound.

Hadron resonance gas model

- Using HRGM + $T > 0$ lattice data to find evidence for new hadrons 1008.1747.
- Plot below from 1207.7287. The hadron resonance gas model: thermodynamics of QCD and Polyakov loop.

![Graph](image-url)

Implications of heavy glueball results from lattice QCD for the
Recent unquenched glueball calculation

- A long time ago we (a group in Liverpool) wanted to study flavour singlet mesons with quantum numbers: \(0^{++}\) and \(0^{-+}\) with our allocation on the QCDOC.
- The ASQTAD improved staggered fermion action with the one loop tadpole improved gauge action was used. (Same as MILC collaboration)
- The strange quark mass was not very well tuned. The parameters were chosen to be part of a larger (but with lower statistics) set of calculations performed by the MILC collaboration (arXiv:0903.3598 for an overview).

<table>
<thead>
<tr>
<th>a (fm)</th>
<th>(m_\pi) (MeV)</th>
<th>L (fm)</th>
<th>Number of configs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>280</td>
<td>2.9</td>
<td>5000</td>
</tr>
<tr>
<td>0.09</td>
<td>360</td>
<td>2.9</td>
<td>3000</td>
</tr>
</tbody>
</table>

The higher statistics allowed to find a signal for the pseudoscalar \(0^{-+}\) glueball.
Variational methods

Measure the correlation between different operators

\[ \tilde{C}_{\alpha\beta}(t) = \sum_{\tau} \langle 0 | \Phi^\dagger_{\alpha}(t + \tau) \Phi_{\beta}(\tau) | 0 \rangle . \]

\[ \tilde{C}(\bar{t}) = \tilde{C}^{-1}(0) \tilde{C}(\bar{t}) . \] (1)

These interpolating operators are a linear sum of the basis vectors:

\[ \tilde{\Phi}_{\alpha\alpha}(t) = \sum_{\alpha} v^i_{\alpha} \Phi_{\alpha}(t) . \]

The mass of the state is extracted by cosh fits of the correlation matrix in the optimal basis:

\[ \tilde{C}_{ii}(t) = |c_i|^2 \cosh (m_i t - N_T/2) , \]

where \( N_T \) is the length of the lattice in the time direction.
Glueball versus expt, (plot from quark model review in PDG, 1005.2473)

- Unquenching effects seem small – but still need continuum limit
- Solid black are quenched, blue unquenched.
Extending the variational basis

- Lucini, Rago and Rinaldi developed a glueball variational code and applied it to large $N_c$ (1007.3879).
- Much bigger basis of glueball operators used.
- Variational basis included two body glueball states.
- Basis included bi-torelon operators (products of two loops winding in a compact spatial direction).

Lucini et al. ran their code on the “UKQCD” staggered configurations with $a = 0.09$ fm. The final publication was 1208.1858.
A comparison of lattice results with PDG and results from Crystal Barrel collaboration. Need 550,000 configurations to get 50 MeV errors on the spin exotic $0^{+-}$ glueball.
Exotic spin quantum numbers

The naive quark model predicts the $P$ and $C$ of a $\bar{q}Γq$ meson with spin $S$ and orbital angular moment $L$

\[
C = (-1)^{L+S}
\]
\[
P = -(-1)^L
\]

Exotic quantum numbers $J^{PC} = 1^{--}, 2^{++}, 0^{+-}, ..$

- Anything you find with exotic $J^{PC}$ is not a bound state of quark-antiquark!!! Write press release!
- Morningstar and Peardon found exotic $2^{+-}$ and $0^{+-}$ glueballs, but they noted other spin assignments are possible.
- It is better to first experimentally look for spin exotic glueballs.
Spin identification

- The introduction of the lattice breaks the rotational symmetry and this has implication for spin identification.
- Unfortunately this is particularly important for hybrids and glueballs with exotic quantum numbers and higher spin states.
- The theory of finite groups is well developed and there are well defined methods to do calculations.
- Parity and charge conjugation as well.
Spin indentification

- The symmetry group of a 3D lattice is the octahedral group.
- There are 5 representations of the octahedral group with names: $A_1$, $A_2$, $E$, $T_1$ and $T_2$, with dimensions: 1, 2, 3, 3.
- The lattice interpolating operators are in a definite lattice representation.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$E$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table:** Subduced representations $J \downarrow G_O$ of the octahedral group up to $J = 4$. This table illustrates the spin content of the irreducible representations of $G_O$ in terms of the continuum $J$. 

Implications of heavy glueball results from lattice QCD for the
Continuum extrapolation PC=++ channel

- The plot from Morningstar and Peardon (hep-lat/9901004) shows glueball operators in different representations as a function of square of lattice spacing.
Meyer and Teper’s calculation

- Meyer and Teper (hep-lat/0306019, hep-ph/0409183) wanted to study higher spin glueballs to study Regge trajectories.
- They used the original Wilson gauge action.
- They used techniques beyond basic group theory to assign operators to spins.
- The string tension was used to set the scale (later $r_0$ was used.)
- Used isotropic lattices but a clever method to reduce the signal to noise ratio.
- Lattice spacings in the range of: 0.1 to 0.05 fm.
- The tables of glueball masses were actually published in Meyer’s thesis hep-lat/0508002.
Comparison of glueball spectrum from lattice

- From review by Mathieu et al. (arXiv:0810.4453)
- Comparison between Morningstar and Peardon (circles) and Meyer and Teper (triangles)
- Recall the exotic quantum numbers: $2^{++}$ and $0^{+-}$
4D SU(3) PC=++ glueballs and isosinglet mesons

\( t = \frac{M^2}{2\pi \sigma} \)
Glueballs as a theory playground

- Perhaps you are not interested in the experimental search for glueballs and would rather think about theoretical issues.
- I sort of believe that it is useful to try and understand the glueball in terms of degrees of freedom.
- There is a good review of glueballs by Mathieu et al. (arXiv:0810.4453).
Trying to understand the glueball spectrum

There have been a number of different attempts to understand the glueball spectrum, but with different physical pictures.

**Massive gluons** There are claims that a gluon mass can be generated dynamically. For example: Cornwall and Soni (Phys.Lett. B120 (1983) 431) developed a model of massive gluons interacting via a potential. Glueballs are then two or three massive gluons interacting.

**Bag models** The basic idea is that for some magical reason the inside of a hadron is perturbative.

**String theory** This was popular a few years ago. The calculations were done in the strong coupling large $N_c$ limit.

**Flux tube model** Used to be agree with lattice for light $1^{-+}$ exotic hybrid mesons.

**Instanton liquid model**

I don’t include calculational methods such as sum rules.
Glueballs from two massive gluons

Comparison of lattice glueballs (circles) with model of glueball with two gluons (squares and triangles). From review by 0810.4453, Mathieu et al.
The hadron spectrum collaboration (1201.2349) claim that the bag model best explains the hybrid spectrum of mesons and baryon.
Glueballs from ADS/CFT and lattice

From review by 0810.4453, Mathieu et al.

Implications of heavy glueball results from lattice QCD for the
Conclusions

- Lattice calculations of glueballs in full QCD are challenging, because the glueball interpolating operators will mix with two meson states, and flavour singlet mesons.
- The PANDA experiment will look for heavy glueballs in 2018.
- It is not clear that looking for heavy glueballs will be easier than for light glueballs, but we can hope.
- There are disagreements between lattice QCD calculations for the masses of the $2^{+-}$ and $0^{+-}$ spin exotic glueballs.
- It is not clear how useful it is to use models to explain glueball spectrum. Finding the effective degrees of freedom is useful, but not if we end up with a number of inconsistent pictures.
Implications of heavy glueball results from lattice QCD for the
Glueballs as a theory playground

Teper and collaborators (hep-lat/0103027) studied glueballs for different $SU(N)$ groups.

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.37(15) + \frac{1.93(85)}{N^2}$$

The masses of glueballs are good test of theoretical formalisms, such as various string theory inspired methods. Some new methods produce results in the strong coupling limit, so it is (perhaps) interesting to compare with old lattice results done using a strong coupling expansion $\frac{1}{g^2}$.

<table>
<thead>
<tr>
<th>Group</th>
<th>Method</th>
<th>$m_{2^{++}}/m_{0^{++}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>strong coupling lattice black hole</td>
<td>1.2 - 1.25</td>
</tr>
<tr>
<td></td>
<td>Klebanov-Strassler</td>
<td>1.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.37</td>
</tr>
</tbody>
</table>

For the supergravity numbers I used the review by Caceres, Journal of Physics Conference 24 (2005) 111.